

# Open problems in polyhedral combinatorics

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E. Egerváry 1931: min-max formula for maximum weight perfect matching in a complete bipartite graph

- first result in *polyhedral* combinatorics

T.E. Easterfield 1946:  $O(2^n n^2)$  algorithm for optimum assignment

J. Robinson 1949: cycle cancelling method for optimum assignment

H.W. Kuhn 1955: Hungarian method

- first polynomial-time method
- primal-dual
- iterative shortest path method

# Open problems in polyhedral combinatorics

$NP \neq P?$

Can a maximum flow be found in  $O(nm)$  time?

Is the matching polytope the projection of a polytope with a polynomial number of facets?

Woodall's conjecture (1978): The minimum size of a directed cut in a directed graph is equal to the maximum number of disjoint directed cut covers.

Give a combinatorial polynomial-time algorithm finding a maximum stable set in a perfect graph.

Is there a polyhedral extension of Mader's  $T$ -paths theorem?

Let  $G$  be an undirected graph and let  $T \subseteq VG$ .

A  $T$ -path is a path connecting two distinct vertices in  $T$ .

Mader 1978:

maximum # of internally disjoint  $T$ -paths = minimum of

...

- polyhedral?
- algorithmic?
- matroidal?