Thesis

Corvinus University of Budapest Eötvös Lóránd University

Big data analysis in financial networks:

An econometric approach for the detection of SIFIs and the measurement of systemic risk

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I Introduction

The last financial crisis shed light on the vulnerability of the financial system. The bankruptcy of Lehman Brothers in 2008 was the signal that dense networks exist in the financial sector, which can pose systemic risk for the whole system. The banks were identified as the primary transmitters of spillovers, but the bailout of American International Group (AIG) demonstrated that insurance companies could get near to bankruptcy.

Moreover, Nyholm (2012) emphasized that the insurance companies are as important, as banks posing systemic risk, despite the insurance sector remained less analysed. If insurance companies were included in researches, they did not exceed sectoral or sometimes individual-level analysis, which missed providing a full overview of risk structure in the insurance branch, while the banking sector was deeply reviewed.

The importance of banks is easy to understand, thus the banks are "classical" actors of financial markets, which implied an increased interest in the literature. While other entities, e.g. insurers, did not stand in the spotlight. Nevertheless, several similarities and differences can be observed between insurers and banks, which makes a relevant research topic identifying the characteristics of their behaviour in financial networks.

To understand the main similarities and differences, I summarize in short, the classical insurance and banking activities, more precisely the life- and non-life insurance and lending.

The traditional insurance activities mean that the insurance companies take over one part of the whole risk from the policyholders for some insurance premium, and the money is invested in different assets to assure financial background for the future uncertain pay-offs (Insurance Europe 2014, p.10).

The stylized balance sheet of an insurance company helps to understand the structure of the most important assets and liabilities and the possible risk factors. Table 1 highlights that the most significant liabilities of the insurance companies derive from the collected insurance premium (policyholder liabilities), which are reserved by the institutions. To fulfill the liabilities, they invest in different assets represented by fixed assets in the balance sheet.

While in the balance sheets of banks, the dominating items among assets are the loans, and between liabilities the deposits. This division is in accordance with the lending activity of the banks. Moreover, the banks also have liquidity reserve, while the duration of the assets is usually longer than the duration of the liabilities.

Naturally, the balance sheet of insurers and banks contains some equity, which is inevitable for running the business on the market.

However, in practice, the banking and insurance activities are overlapping, thus banks provide insurance products, and insurance companies also utilize financial assets, like unit-linked life insurance products, insurance-linked-securities as cat bonds (Szüle 2015, p.16-17.), which can cause the mixture of existing risk sources at the institutional level.

The risk sources also differ in the two cases. For insurers, the problem should be divided

Balance sheet of insurers		Balance sheet of banks	
Assets	Liabilities	Assets	Liabilities
+Fixed assets	+Equity +Policyholder liabilities	+Liquidity reserve +Loans	+Equity +Deposits

Source: Based on the illustration of Szüle (2015) (Szüle 2015, Insurance Europe 2014, p.15, p.23)

Table 1: Stylized balance sheets of insurers and banks

into two parts, the life-insurers must cover life insurance risk (e.g., longevity, investment). In contrast, the non-life insurers mainly depend on the non-life insurance risk. However, the non-life-insurance risk has a higher proportion in the case of non-life insurers, than the life insurance risk in the case of life insurers (Oliver Wyman and Company 2001, p.23). Although for the life insurers, the asset-liability matching poses a higher risk, thus they must invest long term assets to be able to provide services in the far future.

There are some similarities between the risks of the two branches. Szüle (2015) pointed out that the insurance and banking activities include common features, while both fields depend on external effects, like natural disasters or non-performing loans, which can have a severe negative impact on the profitability and solvency of the institutions (Szüle 2015, p.19). Besides, both institutions use risk preventing mechanisms, like credit rating or medical examination to reduce possible losses.

However, as I have mentioned, the duration of the liabilities can differ significantly. At the same time, the deposits of the banks are easier available for the customers in comparison to the policyholder liabilities. Namely, the former is strictly regulated, and deductions are applied for repurchased or cancelled contracts to avoid anti-selection on the insurance market.

Another common point is, that the financial networks play an important role in both markets, which can be a possible source of systemic risk, as the banks are generally highly interrelated because of their lending activities[1] and money transfers. So, the weak performance of a bank can spill over to other parts of the network. Until, in the case of insurers, the reinsurance activity can create deep linkages, although the reinsurers rather reduce the risk of primary insurers, than jeopardize their solvency (Insurance Europe 2014, p.19).

The comparison highlighted that the risks originating from traditional business activities are not homogenous in the case of insurers, while life and non-life insurers face other types of risks, or the structure of risk can be different. Despite this finding, as far as I know,

[1] Two main theories describe the mechanism of lending: endogenous money theory and financial intermediation theory. According to the endogenous money theory, banks are money creators, and their transactions can increase the importance of liquidity management and pose higher systemic risk. For more detail, see the publication of Jakab and Kumhof (2015).

nobody analysed the connections between insurance branches granularly compared with banks. Only as a whole industry or separate groups, like reinsurers, life and non-life insurers were involved in scientific research[2]. Also, I am aiming to understand the inter-sectoral linkages of the insurance industry and its relationship with the banking sector and identifying the critical institutions providing a detailed overview of the structure of the insurance market and its relationship to banks.

The analysis is based on the Granger-causality approach measuring the directional relationship between companies, branches and sectors.

For the individual-level analysis, I applied the modified framework of Hué et al. (2019) proposed by Song and Taamouti (2019), which based on pairwise Granger-causalities among firms, including the first principal component calculated from the remaining data (excluding the original dependent and independent variables). This method makes it possible to reconstruct the real financial network filtering out spurious and indirect linkages, which is a considerable deficiency of several methodologies.

Applying the method of Song and Taamouti (2019) showed a different ranking of systemically important financial institutions, but similarly, the widely used frameworks cannot conclude on a clear order of vulnerable companies.

However, the sectoral level analysis resulted that the number of real connections dropped dramatically during the financial crisis, which was never measured according to my best knowledge. This means that during distressed periods the amount of non-real connections increases, while the real linkages diminish.

The sectoral level investigation presented that the banks with high market capitalization had a central role in the network. At the same time, small banks can also become SIFIs, but their group did not pose a high risk, only some individual institutions.

As far as the insurance industry is concerned, the North American insurance sector was more interconnected than the European one. In the pre-crisis period, the North American P/C insurers were the most significant sector. Still, after the turmoil, the importance of European and North American life insurers have grown.

The structure of the document organized as follows. First of all, I overview the definition and main characteristics of systemic risk in section II, after that part III summarizes the main findings of the literature in order to reveal relevant questions and methods and discuss the deficiencies and merits of the literature. In section IV, I describe the used methods and data for the investigation, the forthcoming part (V) shows the results of the econometric analysis. VI part summarizes the main findings and suggests the path of future research. Last but not least, I conclude the empirical study in section VII.

[2]According to my best knowledge, only Kaserer and Klein (2019) distinguished more separate groups in the insurance sector, but their analysis also missed investigating the sectoral level connectedness.

II Systemic risk

Understanding the basic concept between risk and uncertainty is a fundamental step to define systemic risk. The difference between risk and uncertainty is statistical measurement. The former one is gaugeable, while the later is not (Medvegyev 2011). Nevertheless, the statistical measurement of risk is complicated because it is a latent variable, so it can only be approximated (Kovács 2011).

The systemic risk is hard to define (Benoit et al. 2017), so it is not surprising that the first time the European Central Bank only highlighted the concept of systemic risk (European Central Bank 2009, p.134). Also, systemic risk occurs via an intense systemic event that effects important institutions and markets.

The International Monetary Fund's Financial Stability Board gave a more sophisticated characterization including the depiction of effects of a crisis (International Monetary Fund et al. 2009, p.2): systemic risk is "a risk of disruption to financial services that is (i) caused by an impairment of all or parts of the financial system and (ii) has the potential to have serious negative consequences for the real economy."

While the Group of Ten uses a broader approach for systemic risk. According to the Report on Consolidation in the Financial Sector "systemic financial risk is that an event will trigger a loss of economic value or confidence in, and attendant increases in uncertainly about, a substantial portion of the financial system that is serious enough to quite probably have significant adverse effects on the real economy. Systemic risk events can be sudden and unexpected, or the likelihood of their occurrence can build up through time in the absence of appropriate policy responses" (Group of Ten 2001, p.126).

The formerly mentioned designations presented that several depictions exist for systemic risk. Eling and Pankoke (2014) reviewed 26 definitions from them and identified three major elements (Eling and Pankoke 2014, p.2-3.):

- Risk of an event: there exists an associated event, which derives from the dysfunction, institutional default, or an economic shock.
- Impact of the event: the consequences of the risk to the real economy.
- Causation of the event: the source, which implies the risk.

This framework characterizes the main points of the phenomenon, which helps to recognize the systemic risk. Also, systemic risk means the disruption of the financial system, which can derive from such events, like malfunction of the market actors or deficiencies of the supervisory authority, and causes losses in the real economy or destruction of confidence on the market, while the effects have to reach almost all market players.

However, the mentioned definitions do not interpret the source of the risk deeply. The work of Benoit et al. (2017) is valuable for this reason, thus they divided the systemic risk literature into two parts to specify the sources of systemic risk. The first part is the source-specific approach, which collects qualitative modells and aims to support macroprudential supervision, while the second part includes the global measures of systemic risk (Benoit et al. 2017, p.110).

Benoit et al. (2017) analyse the source-specific approach, and distinguish three major part among these papers (Benoit et al. 2017, p.110,117):

- Systemic risk-taking: why institutions are inclined to be exposed to similar risk factors.
- Contagion: losses spillover from one financial institution to others.
- Amplification mechanism: explaining why small shocks can end up having impacts.

While the source-specific approach focuses on the lower granularity of the network, the global measures try to grasp systemic risk without dissolving small components but manage all risk as a whole system.

My research only focuses on the contagion risk from source-specific aspects in order to understand the mechanism of risk transmission in financial networks. Furthermore, this attribute can be quantified the most precisely by econometric tools. I also utilize global measures, which can express the state of the network by one number.

After considering the theoretical concept of systemic risk, section III will present the relevant papers of the literature.

III Literature review

The bankruptcy of Lehman Brothers was the signal of the financial crisis deepening a decade ago. After that, financial network analysis became the centre of interest, and several studies were published. Silva et al. (2017) analysed 266 articles related to systemic risk, and it was found that the vast majority of researchers focused on banks, while other topics, like the insurance companies, are less researched. This fact motivated me to dive into the analysis of interconnectedness in the insurance sector. I intend to provide a detailed overview of the financial network literature, highlighting the most important domains. I start the review in the III.2 section with research focusing on individual institutions. After that, I summarize the studies highlighting the importance of system dynamics. Last but not least, I focus on such analysis, which takes into consideration both network dynamics and the role of unique institutions.

III.1 Classification of the literature

There exist several approaches to analyse the interconnectedness of financial networks. Bisias et al. (2012) compared numerous measures and frameworks and differentiated four main ways. The researcher can follow a data-driven approach and analyse public and/or private data (Hué et al. 2019, p.87). Another differentiation can be based on the aim of the supervising authority. Two main approaches exist, which use this interpretation of the literature, the macro- and microprudential levels. The former focuses on the behaviour of the whole market, the latter prefers to understand the risk sources of the individual institutions Eling and Pankoke (2014). Furthermore, the time horizon of the event or decision can be a grouping variable. Also, the analysis can focus on pre-event, contemporaneous, and post-event risks, which can provide signals for the supervising authority (Bisias et al. 2012, p.21). Last but not least, the research method is also an important field of systemic risk analysis, thus several measures exist, which try to analyse and forecast the movements of the shocks.

In general, the micro- and macroprudential classification is applied, when the research methods are differentiated (Rodríguez-Moreno and Peña 2013, Eling and Pankoke 2014). Rodríguez-Moreno and Peña (2013) classified the Libor spread, first principal component and CDS indexes as macroprudential metrics (Rodríguez-Moreno and Peña 2013, p.1821), while systemic risk indexes (elaborated by Lehar (2005)), multivariate densities and aggregate co-risk as microprudential gauges.

Eling and Pankoke (2016) also followed this framework and categorized bankbeta, lower tail dependence (LTD) and Granger-causality as macro-level indicators, as ΔCoVaR, MES, Long-run MES (LRMES), DIP and SRISK as micro-level metrics (Eling and Pankoke 2016, p.253-254).

A similar division was proposed by Zhang et al. (2015), who described the first group of indicators, which "captures risk spillovers from a financial institution to the rest of the financial system" (Zhang et al. 2015, p.1405). The ΔCoVaR and ΔACoVaR are the representations of this category. Nevertheless, the second tries to "quantify the degree of vulnerability of financial institutions" (Zhang et al. 2015, p.1405). Two examples are EXSHORT (elaborated by Lehar (2005)) and SRISK. So, the description and the examples show the similarity of the two approaches.

Wang et al. (2018) emphasized another three categories of systemic risk measures. The first one is based on the correlation between variables. So, the principal component-based methodologies belong to this set, like the framework introduced by Billio et al. (2012) or the absorption ratio proposed by Kritzman et al. (2011). The second type of measures tries to quantify systemic risk spillover across financial institutions, like MES, CES, SES, SRISK and ΔCoVaR. Both approaches summarize the interaction of individual institutions and the financial network (Wang et al. 2018, p.2). On the contrary, the third approach focus on network theory and its application on the financial network, as the Dynamic Causality Index (Billio et al. 2012), realized systemic beta (Hautsch et al. 2014) and the tail event-driven network (Härdle et al. 2016, TENET).

Last, but not least Giglio et al. (2016) also proposed a classification including four groups (Giglio et al. 2016, p.460). The first one catches the institution-specific risk expressed by CoVaR, ΔCoVaR, MES, and its modification MES-BE. The second set describes comovement and contagion between financial institutions, like the absorption ratio (Kritzman et al. 2011), DCI (Billio et al. 2012) and Diebold-Yilmaz spillover index (Diebold and Yilmaz 2009, 2014, 2015). The gauges in the third class measure the volatility and instability, like the CatFin (Allen et al. 2012). The fourth group includes indicators about the liquidity and credit environment of the financial system. For example, the illiquidity measure, the difference of the LIBOR and T-bill spread, the default spread or the credit spread (Giglio et al. 2016, p.460).

Every way of classification relies on the granularity of the data expressed by the micro- and macroprudential categories. So it seems to be a reasonable way to split the literature according to the level of the analysis. I follow the first way proposed by Benoit et al. (2017). Also, I review articles reflecting on the individual institutions, the whole network and both of them. Furthermore, I highlight the most important research topics during the overview.

III.2 SIFIs

After the financial crisis, one of the hottest topics was classifying the systemically important financial institutions (SIFIs) and characterizing the main systemic risk contributing factors. These researches appeared in the banking industry (International Monetary Fund et al. 2009) and later also in the insurance branch (International Association of Insurance Supervisors 2011). The first part of the literature mainly focuses on these questions at the individual level.

Chang et al. (2018) investigated individual Taiwan life- and non-life insurers using MES, SRISK and ΔCoVaR methodologies and panel regression to identify important institutions in the period 2005 – 2015. The authors found that the majority of risky insurance companies are working in the life insurance field. The main determining factors of systemic risk are non-core activities and leverage, but the size of the company does not matter. The three measures caught different part of the systemic risk, while MES was almost constant during the analysed period. SRISK indicated the end of the crisis adequately, while ΔCoVaR peaked before the financial crises in 2008 escalated.

Chen and Sun (2019) tried to identify the Global Systemically Important Institutions (G-SII) using network theory to analyse 157 global insurers between 2006 – 2015. The authors developed their own measures, Systemic Risk Receiver and Systemic Risk Emitter (SRR and SRE), and compared them to the traditional measures, like MES and ΔCoVaR. The results demonstrated a quite strong rank correlation between widely used and the author's gauges. The time-varying network also focused on the size and the tail risk of the insurance companies. The results confirmed the importance of G-SII but also pointed out that some non-G-SII can be as dangerous as the G-SII.

113 commercial banks, insurers, asset managers and brokers/dealers were reviewed by Nucera et al. (2016) on the EU market in the period of 2002 – 2013. Principal component analysis, MES, SRISK, ΔCoVaR, Leverage ratio, Dollar Systemic risk and VaR were compared, and its mixture to investigate changing rankings of systemically important institutions. The simple combined metric was proved more stable, than individual indicators.

Nyholm (2012) tested the connections between 21 insurance companies and 31 banks applying Granger-causality and CES on EU data from 1995 – 2014. The data showed that the sectors are highly interconnected with each other, but did not have any difference, as far as, the extreme risk is concerned. Some variations could derive from different financing schemes of the two branches, which means that insurers had long term liabilities, as in the portfolios of the banks, the short term ones were dominating. Although, during turmoils, the different funding horizons did not cause different behaviour for any sectors.

The specifications of the banking and insurance sectors were compared by Weiß and Mühlnickel (2014). However, the empirical analysis concentrated only on insurance companies from the period of the financial crisis (2007 – 2008). Totally, 89 US life, non-life and reinsurers were analysed using MES, ΔCoVaR and SRISK. Weiß and Mühlnickel (2014) confirmed that the insurers also contribute to systemic risk—the vulnerability of insurers derived from non-traditional activities. Furthermore, the size is the most determining factor of systemic risk, but global diversification, short-term funding and substitutability did not pose systemic risk.

Articles	Market players	Classification of insurers?	Answered question(s)	Location	Horizon of the analysis	Frequency of data	Number of assets	Methods
Weiß and Mühlnickel (2014)	banks and insurers	Yes	1, 7	USA	2007 – 2008	daily, annual	totally 89 life, non-life and reinsurers	MES, ΔCoVaR, SRISK, regression
Nyholm (2012)	banks and insurers	No	5	EU	1995 – 2014	weekly	21 insurers, 31 banks	Granger-casuality, CES
Chen and Sun (2019)	banks and insurers	No	1, 3	Globe	2006 – 2015	weekly	157 insurers	MES, ΔCoVaR, Condtional tail loss, SRR, SRE, Lasso, quantile regression
Chang et al. (2018)	banks and insurers	Yes	1, 2, 3, 7	Taiwan	2005 – 2015	daily, quarterly	10 life insurers and 10 non-life insurers	MES, SRISK, ΔCoVaR, panel regression

Notes: [1] Which companies are systemically important financial institutions?, [2] What are the main characteristics of systemically important financial institutions?, [3] Do the systemically important financial institutions change over time?, [5] Is there any existing relationship between banks and insurance companies?, [7] Do compare the authors the different measures?

Table 2: Articles focusing on individual institutions

III.3 System dynamics

The individual-level analysis does not provide an overview of the macro-level changes, which can suddenly change the riskiness of the institutions. On the other hand, besides the macroprudential purposes, the micro-level analysis does not provide enough information for the regulating authority. So, it is not surprising that a significant part of the articles tries to describe the movement of the financial networks highlighting risk amplification factors and possible measures of it.

III.3.1 Dynamics of the financial sectors

Usually, banks, insurers and other financial sectors are investigated at the same time. Still, a few papers concentrate only on one industry, like the first three papers, which depict the banking sector characteristics.

Abendschein and Grundke (2018) analysed 80 North American and European banks to explain the relationship of the real economy and the financial sector in the post-crisis period (2009 – 2016). The widely used MES, SRISK and ΔCoVaR were applied by the authors. They specified the relationship between measures using rank correlation, which showed a low-level connection. The authors identified the rank correlation determining factors, and the size was found not significant, while the leverage had a negative impact on the relationship. The macro variables were not influential in the research, only the country-specific returns had an effect on the rank correlation.

Rodríguez-Moreno and Peña (2013) also focused on geographically separated banks in the USA and Europe between 2004 and 2009. 20 European and 13 US banks were in the sample, and several methods (Granger-causality, first Principal component, multivariate density, Co-risk, LIBOR spread etc.) were compared to find the most precise macro and micro level metric. The measures were compared according to the efficiency of creating ranking from systemically important institutions. The first principal component performed the best among macroprudential gauges, while the multivariate density at the micro-level.

In spite of the previous papers, Castro and Ferrari (2014) concentrated only on 26 European banks in the period of 1999 – 2012, and used Δ CoVaR framework to calculate systemic risk contribution, and developed a modified Kolmogorov-Smirnov-test to measure the significance given by Δ CoVaR. They found it challenging to rank banks according to their contribution to systemic risk. Still, the statistical support of rankings is a worthy contribution from the authors because using their test statistics, the researchers can dismiss the unreal relationships.

The following two papers focus on the insurance network. The Dutch insurers were analysed by van Lelyveld et al. (2011), who divided the insurance sector into three main groups: 77 life insurers, 238 non-life insurers and 350 reinsurers were represented in the sample. The data derived before the financial crisis (2003 – 2005) and quite a large sample was chosen.

Articles	Market players	Classification of insurers?	Answered question(s)	Location	Horizon of the analysis	Frequency of data	Number of assets	Methods
Wang et al. (2017)	banks and insurers	No	4, 5	USA	2006 – 2015	daily	20 banks, 15 insurers, 22 real estate companies, 27 diversified financials	Granger-causality, CaViaR
Cummins and Weiss (2014)	banks and insurers	Yes	2, 5, 6	USA	1988 – 2012	annual	Aggregated data, unique data not available	balance sheet analysis
Bierth et al. (2015)	insurers	Yes	2, 4, 7	Globe	2000 – 2012	daily, monthly, quarterly	112 life insurers, 141 non-life insurers	MES, ΔCoVaR, SRISK, CATFIN, panel regression
Korobilis and Yilmaz (2018)	banks and insurers	No	4	USA and EU	2005 – 2016	daily	8 commercial banks, 5 investment banks, 2 mortgage companies, 1 credit card company and 1 insurer (USA), 17 European banks	DY spillover index, TVP-VAR
Szüle (2019)	banks and insurers	No	5	Hungary	2003 – 2015	annual	16 banks, 17 insurers	absorption ratio, MDS
Dreassi et al. (2018)	banks and insurers	No	2, 4, 7	EU	2006 – 2014	weekly	21 banks, 9 insurers	correlation, factor modell
Adams et al. (2014)	banks and insurers	No	4	Global	2003 – 2010	daily	8 investment banks, 26 commercial banks, 31 insurers, 47 hedge funds	SDS-VAR, PCA, quantile regression
Kleinow et al. (2017)	banks and insurers	No	7	USA	2005 – 2014	daily	49 banks, 30 insurers, 43 non-depository financial institutions	Co-Risk, MES, ΔCoVaR, LTD

Notes: [2] What are the main characteristics of systemically important financial institutions?, [4] What are the characteristics of system dynamics?, [5] Is there any existing relationship between banks and insurance companies?, [6] Is there any existing relationship between interconnectedness and the macroeconomy?, [7] Do compare the authors the different measures?

Table 3: Articles focusing on individual institutions and financial networks I.

Articles	Market players	Classification of insurers?	Answered question(s)	Location	Horizon of the analysis	Frequency of data	Number of assets	Methods
Elyasiani et al. (2016)	banks and insurers	No	5	Globe	1991 – 2012	daily	52 banks, 43 insurers	GARCH
Diebold and Yilmaz (2015)	banks and insurers	No	4	USA and EU	2004 – 2014	daily	7 banks, 1 insurer, 5 investment banks, 2 mortgage companies, 1 diversified financial company, 1 credit card company (USA), 18 European banks	DY spillover index
Bernal et al. (2014)	banks and insurers	No	5	USA and EU	2004 – 2012	daily	banks, insurers, financial service companies	Kolmogorov-Smirnov test, ΔCoVaR, quantile regression
Castro and Ferrari (2014)	banks	-	1	EU	1999 – 2012	daily	26 banks	ΔCoVaR, test of significance, quantile regression
Sedunov (2016)	banks and insurers	No	2, 4, 7	USA	1998 – 2008	daily, quarterly	50 banks, 56 insurers, 50 brokers	ExposureCoVaR, SES, Granger-causality, regression
van Lelyveld et al. (2011)	insurers	Yes	4	Netherlands	2003 – 2005	annual	77 life insurers, 238 non-life insurers, 350 reinsurers	solvency indicator, stress-testing
Zhang et al. (2015)	banks and insurers	No	7	Globe	1992 – 2006	daily	197 banks, 68 insurers, 38 other financial institutions	ΔCoVaR, ΔA_CoVaR, EXShort, SRISK, Grager-causality, logistic regression

Notes: [1] Which companies are systemically important financial institutions?, [2] What are the main characteristics of systemically important financial institutions?, [4] What are the characteristics of system dynamics?, [5] Is there any existing relationship between banks and insurance companies?, [7] Do compare the authors the different measures?

Table 4: Articles focusing on individual institutions and financial networks II.

Articles	Market players	Classification of insurers?	Answered question(s)	Location	Horizon of the analysis	Frequency of data	Number of assets	Methods
Giglio et al. (2016)	banks and insurers	No	4, 6, 7, 8	USA and EU	1946 – 2011 (USA), 1978 – 2011 (UK), 1994 – 2011 (EU)	monthly	20 US, UK and European financial, insurance and real estate companies	PCQR, PQR
Irresberger et al. (2017)	banks and insurers	No	2, 7	Globe	2007 – 2008	daily, annual	148 banks, 98 insurers	Grager-causality, MES, quantile and probit regression, leverage, DGC
Rodríguez-Moreno and Peña (2013)	banks	-	4, 7	USA and EU	2004 – 2009	daily, annual	20 European, 13 US banks	Granger-causality, correlation, CR, GG metric, PCA, risk index, MD, systemic factor, multinomial regression, LS, AΔCoVaR, Aggregated ES
Abendschein and Grundke (2018)	banks	-	2, 6	North America and EU	2009 – 2016	daily, quarterly	80 banks	MES, SRISK, ΔCoVaR, rank correlation, panel regression
Berdin and Sottocornola (2015)	banks and insurers	No	2, 4, 7	EU	1999 – 2013	daily	20 banks, 20 insurers, 20 non-financials	Dynamic-MES, ΔCoVaR, Granger-causality

Notes:
[2] What are the main characteristics of systemically important financial institutions?, [4] What are the characteristics of system dynamics?, [6] Is there any existing relationship between interconnectedness and the macroeconomy?, [7] Do compare the authors the different measures?, [8] Do aggregate the authors the systemic risk measures?

Table 5: Articles focusing on individual institutions and financial networks III.

The vast majority of the articles put emphasis on understanding the dynamics of financial institutions. Nevertheless, some papers are aiming to illustrate possible future changes in financial networks. Stress scenarios were developed by van Lelyveld et al. (2011) to investigate the resistance of the sector. The results are promising. The bankruptcy of the whole reinsurance sector will not cause severe changes in the stability of primary insurers. Still, the occurrence of such event will severely affect the non-life insurance companies, who are more interconnected with the reinsurers.

Instead of reflecting on one country, Bierth et al. (2015) included globally important 112 life insurers and 141 non-life insurers from the period of 2000 – 2012. A wide range of methods was applied in the article, like MES, SRISK, ΔCoVaR and CATFIN. MES, SRISK and ΔCoVaR corroborate that the insurers had a low impact on systemic stability before the financial crisis. Still, their contribution peaked at the end of 2008 and decreased gradually in the next years. Bierth et al. (2015) stated that the insurance companies contribute in a small compass to systemic risk globally. But they could not confirm the hypothesis, that the life insurers pose higher systemic risk than the non-life insurers. The interconnectedness is responsible for the majority of instability caused by the insurance sector, besides the leverage is another important factor.

III.3.2 Dynamics of multi-sectoral financial networks

Other articles analyse more extended financial networks, including both banks and insurers, sometimes other companies, like financial holdings, real estate companies or hedge funds.

Zhang et al. (2015) tested ΔCoVaR, ΔA_CoVaR, SRISK and EXSHORT projecting power on the data of Asian, LTCM and the global financial crisis in 2008. The analysed sample consisted of 197 banks, 68 insurance companies and 38 other globally important financial institutions. The ΔCoVaR was found efficient for all shock forecasting on the sample.

Szüle (2019) investigated the relationship between banks and insurance companies reflecting on the Hungarian market between 2003 and 2015. The research is based on the annual return calculated from the balance sheets of private limited companies, thus public data is not available from this type of companies. The article used the absorption ratio developed by Kritzman et al. (2011) to catch the interconnectedness, and an existing relationship was found. However, the return based risk is less determined in the insurance sector than between banks.

Irresberger et al. (2017) looked for the systemic risk determining factors applying probit regression on 148 banks and 98 insurers data from the pre-crisis and crisis periods. It concluded that size is the most robust factor, but leverage and interconnectedness also have a significant impact on systemic risk. The explaining variables included systemic risk measures, like MES, ΔCoVaR, interconnectedness index based on Billio et al. (2012) framework and leverage. The comparison of explaining the power of MES and ΔCoVaR showed that MES was higher for insurers, while ΔCoVaR behaved on the contrary.

The research of Elyasiani et al. (2016) also based on global financial institutions, more precisely 52 banks and 43 insurers. The authors tried to reveal the connection between the sectors using GARCH model in the case of mergers for quite a long period (1991 – 2012). The results pointed out that the potentially acquired banks and insurers experience positive abnormal returns, after the announcement of mergers. The abnormal effects seemed to be more long-lasting in the case of insurers compared to banks. However, in the opposite case, only the banks experience positive excess return (Elyasiani et al. 2016, p.712).

Bernal et al. (2014) aimed to characterize the most dangerous sectors to the system stability in Europe and the USA, so they used daily returns of banks, insurance companies and financial services to calculate ΔCoVar and Kolmogorov-Smirnov test confirming its explanatory power. They found different branches relevant in various markets between 2004 and 2012. In the EU, the other financial institutions had the highest contribution to systemic risk, the second most dangerous sector was banking, followed by the insurers. However, the insurance companies proved the riskiest in the USA while the banks the safest.

Cummins and Weiss (2014) used balance sheets of US banks and insurers exploring the sectoral differences in the period of 1988 – 2012. It highlighted that maturity mismatch is a more common problem in banking than in insurance (Cummins and Weiss 2014, p.500). At the macroeconomic level, the banks posed higher risk, than insurers. Still, life insurers were more vulnerable to inter-sectoral shocks, but both life and property and casualty insurers are exposed to the risk of the bankruptcy of the reinsurer network. The primary sources of systemic risk were found the non-traditional activities, like financial guarantee insurance and derivative trading.

Adams et al. (2014) elaborated a new measure called state-dependent sensitivity VaR (SDSVaR), which takes into consideration the "mood" of the financial market. The authors found evidence that small shock can be more harmful during volatile periods because the spillover effects are faster than in regular times. The riskiest part of the economy was the banks and the hedge funds according to the SDSVaR on the analysed sample and time period between 2003 and 2010, which consisted of 8 investment and 26 commercial banks, 31 insurance companies and 47 hedge funds. Unfortunately, this methodology can prove the existence of risk, but cannot characterize the sources of it. Due to this recognition, the network-based literature also tries to reflect on the most important factors.

One of the most widespread ways to determine the evaluation of financial networks is the simple ranking of the institutions. Berdin and Sottocornola (2015) presented a ranking based approach taking into consideration Dynamic-MES, ΔCoVaR and Granger-causality network, and demonstrated that banks dominate the rankings in the EU among 1999 and 2013, but some insurers have also place in it. However, they found no evidence on the higher systemic risk contribution of the insurance industry (Berdin and Sottocornola 2015, p.14). The systemic risk, caused by the insurance sector, was dominated by non-traditional activities. Moreover, the diversification in investments increased the systemic risk, but the role of the leverage is hard to characterize. They emphasized the size as the most crucial factor proposing systemic risk. Besides, the rankings provided by the three methods showed different results. Also, the contribution to systemic risk depends on the elected measure, thus they grasp different parts of the risk.

Not only the dynamics of contagion is a crucial question, but also the channels, which transfer the spillovers in crisis periods. This topic was depicted by Dreassi et al. (2018), who used excess correlation and factor modell to identify the most dangerous risk transferring ways across European banks and insurance companies in the period of 2006 – 2014. They concluded that asset-holding and guarantee channels jeopardize the stability of the insurance companies, while the collateral channel can cause harm for the banks. Size and income from investments were found as the main factors for insurers, while for banks, the capital adequacy, the financing, and income diversification were highlighted.

In general, volatility spillover in financial networks cover the "mean spillover", but the downside risk plays a subordinate role in the literature, because of this is outstanding the article of Wang et al. (2017), who analysed the extreme risk spillovers on financial networks using CaViaR and Granger-causality. The authors depicted that banks and diversified financial institutions are responsible for transferring tail-risk, while insurance and diversified financials are receivers. Important to highlight that the financial spillover has a time lag, so the network needs time to transport risks and information.

Kleinow et al. (2017) compared MES, CoDependence Risk, ΔCoVaR and Lower Tail Dependence on the data of 49 banks, 43 non-investment financial institutions and 30 insurers from the USA 2005 – 2014.They found the reliability of the different frameworks quite low, so calculating more measures is widely recommended.

Giglio et al. (2016) reviewed 19 different measures and used a mixture of them to predict macroeconomic shocks applying principal component quantile regression and partial quantile regression on the data of the 67 largest US and 60 European financial institutions. The methods were robust in forecasting the distribution of macroeconomic shocks.

Diebold and Yilmaz (2015) compared 18 European and 17 US financial institutions volatility spillover utilizing the DY framework between 2004 – 2014. The US data consisted of 7 banks, 1 insurer, 5 investment banks, 2 mortgage companies, 1 diversified financial institution and 1 credit card company, while the European sample included only banks. The main finding of the research was the dynamics of the crisis between the two markets, which means during the 2008 turmoil, the contagion was transferred from the US financial institutions to the European ones, to the contrary, the European sovereign crisis induced a reverse spillover.

Sedunov (2016) tried to identify proper systemic risk forecasting metrics. The authors contrasted Adapted Exposure CoVaR, SES and Granger-causality and attempted to specify the best one for forecasting. Sedunov (2016) compared the models on the US data set consisted of 50 banks, 56 insurers and 50 brokers from the period of 1998 – 2008. The ECoVaR was only appropriate for projecting systemic risk. Besides, institution size, foreign equity exposure and securitization income were concluded as systemic risk contributing factors.

Korobilis and Yilmaz (2018) compared the TVP-VAR methodology with the rolling window based version of the DY framework on 2005 – 2016 USA and EU institutions. In detail, 8 commercial banks, 5 investment banks,2 mortgage companies, 1 credit card company and 1 insurer and 17 European banks were included in the data. The DY spillover index completed with TVP-VAR could better identify the turning points of the financial crisis and resulted more accurate forecast.

III.4 Combined analysis

Several articles made complex analysis taking into consideration the behaviour of both individual institutions and networks. New questions raised by these articles are the measuring of changing SIFIs over time, the quantification of cross-sectoral effects between banks and insurance companies and the connection between real economy and interconnectedness.

III.4.1 Insurance networks

I divide the papers according to the type of analysed institutions. Firstly, I review the articles focusing only on insurance companies, after that the banking sector-specific ones, lastly, the papers reflecting on the extended financial sector.

Chen et al. (2013) divided the insurance sector to 3 main branches: life-non-life and credit risk insurers (CRIs). The credit risk insurers were deeply analysed splitting them into financial guarantee insurers and CDS trading insurers. The whole sample consisted of 17 life insurers, 77 non-life insurers and 20 credit risk insurers (12 credit risk insurer and 8 CDS trading insurers) from North America in the period of 2001−2011. The applied methods were bankbeta, Riskinv, MES and leverage and panel regression to identify significant systemic risk determining factors. The authors concluded that traditional underwriting activities do not pose systemic risk for the insurance sector. In contrast, the non-traditional activities managed by the credit risk insurers are risky, mostly the collateral insurance. Furthermore, Chen et al. (2013) concluded that the downgrading of credit risk insurers increased the credit spread indicating growing systemic risk.

An extensive data set was investigated by Chen et al. (2018), who collected 3012 US property and casualty insurers and 9190 non-US regulated reinsurers from the first one and a half-decade of the 21^{st} century. Primarily, the network-based approach stood in the focus of the authors, especially network density, network centrality, survival and loss given default. The consequences of the bankruptcy of the 10 biggest insurers would be not dangerous. At the same time, the interconnectedness of the insurance network was not high, so it would not cause the crash of the insurance sector.

III.4.2 Banking networks

Some authors shed light on the two-fold dimensionality of the banking sector, the individual effects and the contagion spillover between banks.

The relationship among the economy and the globally important banks was researched by Corsi et al. (2018), who analysed 33 banks and 36 sovereign bonds applying Granger-causality tail networks. The investigation aimed to identify terms between 2006–2014, which propagates the risk spillover in financial networks. Furthermore, the rating of sovereign bonds was used to find the relevant channels of risk transmission. The European sovereign debt crisis was characterized being an unstable period proposing high risk for the whole financial system. The authors also identified interconnectedness as a significant variable in forecasting sovereign bond ratings, which signaled the relationship between banks and the real economy.

Demirer et al. (2018) also focuses on the relations among sovereign bonds and banks. The authors' data set included 96 banks from several countries and G-7[3] countries bonds from the 2004 – 2014 period. The Diebold-Yilmaz spillover index was complemented with LASSO regression selecting proper time lag for VAR modell. The DY spillover index suggested that the equities had a geographical component, while the bonds do not have. But the interconnectedness of the bonds was increased during the last crisis, which derived from the cross-country effects.

The Diebold-Yilmaz framework was the starting point of the methodology elaborated by Hale and Lopez (2019). The method was compared with other measures on US bank holding companies data between 2002 – 2017. The authors' framework based on the decomposition of gauges extracting the effect of the market using regression. The measure was contrasted with DIP, SRISK and CoVaR. The new measure was found to be appropriate for microprudential risk identification.

Hué et al. (2019) used the leave-one-out-concept to determine global systemically important institutions between 90 banks using Granger-causality. The authors proposed a new measure to quantify the negative returns of banks. Moreover, the ranking of the most and less risky institutions was created for pre-crisis, crisis and post-crisis periods. The American banks were pointed out as the most dangerous in the sample. The size and the business model were the most significant factors contributing to systemic risk.

III.4.3 Insurance and banking networks

Chen et al. (2013) reflected on the relationship between 11 insurers and 22 banks from the

[3]G-7 countries are the USA, Germany, France, Japan, UK, Canada, and Italy.

North American region between 2001 and 2011. The linear and non-linear Granger-causality test assured that the banks had an effect on insurer companies, but not vice-versa, if the data was adjusted with heteroscedasticity.

One fundamental article is written by Billio et al. (2012), who analysed the network of banks, insurers, hedge funds, brokers and dealers and included 25 individual institution from each branch between 1994 – 2008.

The proposed methodology is based on principal component analysis and Granger-causality and separates the caused and suffered risks from other institutions. So, the Degree of Granger-causality is an institution based measure, which aggregates the individual effects on the whole system. According to the DCI, the banking sector played a leading role in transferring risk, while besides banks, the insurance sector is the second most vulnerable branch.

Geraci and Gnabo (2018) also focused on four sectors: 71 banks, 40 insurers, 21 dealer-broker and 23 real estate companies were analysed in the time-varying VAR network. The banks and broker-dealers were found the riskiest institutions, which transfer risks to insurers. The consequence of the research was that the TVP-VAR methodology resulted more stable ranking, like rolling-window based density metrics, while it takes into consideration the tail behaviour of extreme events.

Gong et al. (2019) accomplished the sector level analysis on the Chinese market, reviewing the connections between 15 banks, 2 insurer companies and 7 security companies between 2007 and 2017. The authors applied Granger-causality to determine the systemically important institutions and observed growing connectedness during the turmoils. The interconnectedness was grown dynamically during the crisis. They extracted the market effect from the data to concentrate on the individual risk transfer and concluded that the banks and insurers were highly interrelated. A further solution of the article is that the authors compared their gauge with other ones (like CoVaR, SRISK, MES and AR-DCC GARCH was introduced to express non-linearity in the modells), and showed that they produce a similar outcome in detecting systemic risk.

Lin et al. (2018) gave a complex review about the Taiwan financial system between 2005 – 2014 revealing the relationship between 10 banks, 7 insurers and 14 financial holdings. Several methods were contrasted, like Granger-causality, SRISK, MES, ΔCoVaR and DCC-GJR GARCH to express non-linearity. The authors created rankings from the numeric results to describe the systemically important institutions and their changes over time. The metrics presented similar rankings for unstable institutions. However, a moderate rank correlation was found between gauges. The systemic risk contributing factors at the institutional level were size, leverage, profitability, and solvency.

High-frequency data are available from stock markets, but only a few studies on the topic of interconnectedness and contagion in financial networks utilize the advantage of them.

Articles	Market players	Classification of insurers?	Answered question(s)	Location	Horizon of the analysis	Frequency of data	Number of assets	Methods
Chen et al. (2014)	banks and insurers	No	5	North America	2001 – 2011	daily, intraday	11 insurers, 22 banks	linear and non-linear Granger-causality
Lin et al. (2018)	banks and insurers	No	1, 2, 3, 7	Taiwan	2005 – 2014	daily	10 banks, 7 insurers, 14 financial holdings	Granger-causality, SRISK, ΔCoVaR, DCC-GJR GARCH, MES
Jentsch and Steinmetz (2016)	banks and insurers	No	4	EU	2008	5 minutes data	6 banks and 2 insurers	DY spillover index
Geraci and Gnabo (2018)	banks and insurers	No	3, 4, 5, 7	Global	1993 – 2014	monthly	71 banks, 40 insurers, 21 broker-dealers, 23 real estate companies	TVP-VAR, rankings, Granger-causality
Billio et al. (2012)	banks and insurers	No	4, 5	Global	1994 – 2008	monthly	25 – 25 banks, insurers, hedge funds, broker-dealers	Granger-causality, PCA
Gong et al. (2019)	banks and insurers	No	1, 4, 5, 7	Asia	2007 – 2017	daily	15 banks, 2 insurers, 7 security companies	CoVar, SRISK, MES, AR-DCC GARCH, Granger-causality
Engle et al. (2015)	banks and insurers	No	1, 4, 6	EU	2000 – 2012	daily, quarterly	72 banks , 36 insurers, 53 financial services firms, 35 real estate companies	DCC-GARCH, SRISK, Capital shortfall, long-run marginal shortfall, Granger-causality

Notes: [1] Which companies are systemically important financial institutions?, [2] What are the main characteristics of systemically important financial institutions?, [3] Do the systemically important financial institutions change over time?, [4] What are the characteristics of system dynamics?, [5] Is there any existing relationship between banks and insurance companies?,
[6] Is there any existing relationship between interconnectedness and the macroeconomy?, [7] Do compare the authors the different measures?

Table 6: Articles focusing on financial networks I.

Articles	Market players	Classification of insurers?	Answered question(s)	Location	Horizon of the analysis	Frequency of data	Number of assets	Methods
Demirer et al. (2018)	banks	-	4	Globe	2003 – 2014	daily	96 banks, G-7 sovereigns	DY spillover index, LASSO regression
Chen et al. (2018)	insurers	Yes	4	USA	2000 – 2015	annual	3012 US P/C insurers, 9190 non-US regulated reinsurance counterparties	network density, centrality, LGD, survival
Chen et al. (2013)	insurers	Yes	4, 7	North America	2006 – 2009	daily	17 life, 77 non-life and 20 CRI insurers	Bankbeta, Riskinv, MES, Leverage, panel regression
Diebold and Yilmaz (2014)	banks and insurers	No	4, 7	USA	2007 – 2008, 1999 – 2010	5 minutes, daily	7 commercial banks, 2 investment banks, 1 credit card company, 2 mortgage financial companies, 1 insurer	DY spillover index
Nucera et al. (2016)	banks and insurers	No	1, 3, 7, 8	EU	2002 – 2013	monthly	113 commercial banks, insurers, asset managers and broker/dealers	PCA, MES, SRISK, ΔCoVaR, VaR, Leverage ratio, Dollar Systemic risk
Corsi et al. (2018)	banks	-	6	Globe	2006 – 2014	daily	33 banks, 36 sovereigns	Granger-causality tail risk network

Notes: [1] Which companies are systemically important financial institutions?, [3] Do the systemically important financial institutions change over time?, [4] What are the characteristics of system dynamics?, [6] Is there any existing relationship between interconnectedness and the macroeconomy?, [7] Do compare the authors the different measures?, [8] Do aggregate the authors the systemic risk measures?

Table 7: Articles focusing on financial networks II.

Articles	Market players	Classification of insurers?	Answered question(s)	Location	Horizon of the analysis	Frequency of data	Number of assets	Methods
Kaserer and Klein (2019)	banks and insurers	Yes	1, 4, 7	Globe	2004 – 2014	daily	147 banks, 9 multi-line insurers, 15 life, 14 P/C insurers, 8 bond/mortgage insurers, 8 reinsurers	DIP, Marginal DIP, CoPSD, CoPD
Hautsch et al. (2014)	banks and insurers	No	1, 4	EU	2006 – 2010	daily, quarterly	13 banks, 7 insurers	time-varying systemic risk network, realized systemic beta
Wang et al. (2018)	banks and insurers	No	1, 4	China	2008 – 2016	weekly	3 insurers, 14 banks, 7 securities	dynamic TENET, TC, GE, ISS, OSS, SCS, ISI, OSI
Hué et al. (2019)	banks	-	1, 2, 3, 4	Globe	2003 – 2018	daily	90 banks	Grager-causality, leave-one-out-concept
Hale and Lopez (2019)	banks	-	7	USA	2002 – 2017	daily, quarterly	27 banking holding companies	network density, Midas regression, regression, DIP, SRISK, CoVaR

Notes: [1] Which companies are systemically important financial institutions?, [2] What are the main characteristics of systemically important financial institutions?, [3] Do the systemically important financial institutions change over time?, [4] What are the characteristics of system dynamics?, [7] Do compare the authors the different measures?

Table 8: Articles focusing on financial networks III.

One is written by Diebold and Yilmaz (2014), who put the focus on 5-minutes return and return volatility of 7 commercial banks, 2 investment banks, 1 credit card company and 1 insurer from the period of the financial crisis (2007 – 2008) and also include a longer time span from 1999 to 2010. The Diebold-Yilmaz framework was theoretically contrasted to the widely used MES and ΔCoVaR, but the practical analysis was made by the DY spillover index. The spillover table showed a high connectedness not only during turmoils, but also during tranquil times, and the outstanding connectedness of the largest banks.

Engle et al. (2015) analysed 96 financial institutions (banks insurance companies, financial service companies and real estate companies) using the DCC-GARCH method, Capital Shortfall, LRMES and SRISK. The data derived from the 2000 – 2012 period. The European financial institutions were found to be more vulnerable than the US ones (Engle et al. 2015, p.179). The immediate and the 1 day lagged world return was identified to have the highest impact on the European spillovers. Besides, the authors reflected on the effect of SRISK on the macroeconomy, and they found significant Granger-causality between SRISK and industrial production and business confidence index. Furthermore, the authors provided rankings of systemically important banks and insurers, which almost totally covered the globally important institutions.

Hautsch et al. (2014) applied time-varying systemic risk network on 13 banks and 7 insurers in Europe between 2006 and 2010. The institutions were ranked by realized systemic risk beta, and it was found very volatile over time. During quiet times the network effects were dominating in contribution to systemic risk.

Kaserer and Klein (2019) analysed 147 banks and 54 insurers between 2004 and 2014 and distinguished subgroups of insurers, like multi-line, life, property and casualty, bond/mortgage insurers and reinsurers. The authors quantified several measures, like Distressed insurance Premium (DIP), Marginal DIP, Conditional Probability of Systemic Distress, Conditional Probability of Default. They concluded that the banks were responsible for the majority of systemic risk. In contrast, the insurance sector contributed in a small compass to systemic risk, but there were some systemically important insurers. Mostly, the life and multi-line insurers proposed higher systemic risk. The bond and mortgage insurers were characterized as sensitive firms to economic shocks, but they do not influence the stability of the financial system. A further result was that public companies were found to be more vulnerable in comparison to private companies.

The modified version of ΔCoVaR and the tail-event driven network (TENET) got into the focus of Wang et al. (2018) to describe China's financial market dynamics in crisis and post-crisis times, 2008 – 2016. A bulk of metrics were quantified to signal systemic risk, like total connectedness (TC), global efficiency (GE), in-strength of the sector (ISS), out-strength of the sector (OSS), the strength of cross-sector (SCS), in-strength of the institution (ISI), out-strength of the institution (OSI). A quite small sample was utilized by the authors, including 3 insurance company, 14 banks and 7 securities. The banks were classified as the highest tail risk emitters, followed by securities and insurance companies, and also the top 3 most vulnerable institution were banks. In summary, the large banks and insurers were found to be systemically important institutions, but small companies can pose systemic risk if they are deeply integrated into the financial network.

III.5 Conclusions of the literature

A wide range of related articles exist in the topic of systemic risk, to emphasize the main findings of the papers, I highlight the 8 main topics of the papers.

Firstly, the identification of central agents in financial networks became a crucial research field after the bankruptcy of Lehmann Brothers in 2008. In general, the banks are characterized as systemically important institutions, which cause the majority of systemic risk. However, the insurers are also posed systemic risk (Cummins and Weiss 2014), but the leading role of insurers in the vulnerability of financial networks are geographically changing. For example, Wang et al. (2018) identified large insurance companies systemically important, while Wang et al. (2017) found insurers as absorbers of shocks.

Secondly, lots of papers looked for the determining factors of systemically important financial institutions.

The majority of the articles (Irresberger et al. 2017, Berdin and Sottocornola 2015, Dreassi et al. 2018, Sedunov 2016, Hué et al. 2019, Lin et al. 2018) concluded size as the most outstanding factor explaining the vulnerability of individual institutions. Only Chang et al. (2018) rejected size as a significant explaining variable of systemic risk contribution.

Furthermore, leverage was also found as an important factor according to Chang et al. (2018), Irresberger et al. (2017), Bierth et al. (2015), Lin et al. (2018), while Irresberger et al. (2017) doubt its relevance.

Despite some debates between researchers, the literature is conclusive about the role of diversification of investment income, which unambiguously increases the systemic risk contribution of financial institutions (Berdin and Sottocornola 2015, Dreassi et al. 2018, Sedunov 2016).

In the case of insurers, the non-insurance activities, like collateral insurance, derivative trading etc., pose systemic risk as showed by Chang et al. (2018), Cummins and Weiss (2014), Dreassi et al. (2018), and was demonstrated by the AIG bailout, which possessed a wide range of poisoned assets.

Other aspects were pointed out, like the business modell of insurers, which is highly connected to the classical and non-core activities (Hué et al. 2019) . The interconnectedness was classified as a source of risk by Bierth et al. (2015), while Irresberger et al. (2017) did not agree with this finding. Sedunov (2016) emphasized that foreign equity exposure increases instability, similarly as profitability and solvency showed by Lin et al. (2018).

The changes of SIFIs were presented using the ranking of institutions. The central question was elaborating a methodology, which provides a stable ranking. Chang et al. (2018), Geraci and Gnabo (2018), Hué et al. (2019), Lin et al. (2018), Nucera et al. (2016) also presented a solution to this problem, while Chen and Sun (2019) revealed the problems of rankings, proving that some non-G-SII also can harm the financial system.

Furthermore, the dynamics of the networks were depicted several times, the most obvious statement, that the connectedness of financial networks was growing and peaked during the financial crises, which is not surprising, taking into consideration, that the methodologies were created ex post the turmoil. The "mood" of the market is profoundly influencing the movements showed by Adams et al. (2014), but in turbulent times small shock can lead to high losses thanks to the amplification mechanism (Corsi et al. 2018). The USA dominated the evaluation of the financial markets during the financial crises. Nevertheless, the EU market transferred risk to the USA under the European sovereign debt crisis Diebold and Yilmaz (2015). Dreassi et al. (2018) identified the risk transferring channels, which can help policymakers intervening in the movements of the market.

The fifth central topic focused on the interconnectedness among banks and insurers. The literature concludes that there is an existing relationship between the banking and insurance industry (Nyholm 2012, Szüle 2019, Elyasiani et al. 2016, Wang et al. 2017, Chen et al. 2014, Billio et al. 2012, Geraci and Gnabo 2018, Gong et al. 2019). The general direction of risk spillover is from banks to insurers, but Chen et al. (2013) showed that insurers also could Granger-cause banks.

The sixth question is whether interconnectedness can influence the macroeconomy. The statement is usually confirmed by academic papers, like Abendschein and Grundke (2018) investigated the relationship of banks and sovereign bonds and concluded that country-specific returns are essential in explaining the rankings of systemically important institutions. Corsi et al. (2018) also focused on sovereign bonds, especially its ratings, which could be explained by measuring interconnectedness.

Another aspect was lit by Cummins and Weiss (2014), who showed that the underwriting cycle[4] was more dangerous to property and casualty insurers than the financial crises (Cummins and Weiss 2014, p.506).

Engle et al. (2015) analysed a different topic, the predictability of macroeconomic variables, and SRISK metric was found efficient for this purpose. Giglio et al. (2016) also investigated forecasting and summarized some stylized facts of systemic risk (Giglio et al. (2016), p.466-470):

- Systemic risk highly depends on the downside risk.

[4]The underwriting cycle means the periodicity of the insurance market. Also, in the beginning, a few insurers are on the market. They try to acquire higher market share lowering their insurance premiums, which cause the bankruptcy of some insurance companies and low prices for the customers. After that, the insurance premiums begin to increase gradually, which results higher profitability and attract more insurers entering the market, and the cycle begins from the starting point.

- The volatility of the equities provides information to future movements.
- The shocks of the financial markets project properly the monetary interventions, which are effortless handling extreme downside risk.

These attributes help to depict and project systemically risky events, thus the extreme risk spillovers are determining in systemic risk, so the policymakers should reflect on that kind of change. Furthermore, equity volatility is adequate for empirical research because it contains valuable information for possible measures of systemic risk. Last but not least, the mechanism of the monetary authority can be an informative variable, while the shocks are followed by monetary interventions.

Another important topic is the relevance of the different measures. Rodríguez-Moreno and Peña (2013) stated that the CDS spread is more informative for empirical analysis compared to the balance sheet and volatility data. Moreover, the remark of Hale and Lopez (2019) is worth considering that the effect of the market should be excluded before the further investigation, thus it can make distortions in results.

After considering proper data sources and a best practice, I want to highlight the results of the simple comparison and evaluation of measures. Berdin and Sottocornola (2015), Gong et al. (2019), Lin et al. (2018) identified metrics providing similar results or rankings of SIFIs. Sedunov (2016), Zhang et al. (2015) tried to find proper measures for forecasting and concluded that ΔCoVaR and its variations are efficient in projecting. Despite the similarities, Kleinow et al. (2017) pointed out the inconsistency of systemic risk gauges, so the researchers should apply different metrics. Another way - to improve the results - is using TVP-VAR (Geraci and Gnabo 2018) or principal component analysis, which produces more robust outcomes.

The last set of articles points out the efficiency of combining models, as mentioned above. While the combination contains all advantages of the measures calculated from different data sources, it can also reduce the level of noise, and thus increasing predictive power.

III.6 Discussion of the literature

Comparing a large sample of the literature, I identified three main fields, where the publications generally have some deficiencies, like the dataset, the aggregation level and the methodology. In connection with the analysed data, I have to make some remarks.

First of all, the authors generally investigated the insurance market as a whole and homogenous industry. However, the different branches, like life, health, property and casualty, credit and mortgage insurance and reinsurance were rarely compared to each other. (Except Kaserer and Klein (2019), but their analysis did not treat health insurance as an independent branch.)

Furthermore, some papers neglected the countries, which went bankrupt during their analysed time span, which can cause survival-bias in the results.

Another aspect is also focusing on sample choice, which usually relied on the market capitalization of the companies. This way of institution selection is accepted, while the size was identified as one of the most critical risk factors, nevertheless in exceptional cases demonstrated by Wang et al. (2018) the small companies can pose systemic risk, which are usually omitted from the data set.

Two technical terms can have a negative influence on the results of the research. One hand, the publicly traded companies, provide proper data for high-frequency analysis, while private firms do not. Nevertheless, Szüle (2019) showed a method, which is suitable for private companies and Kaserer and Klein (2019) integrated both types of institutions in their research.

The next topic is the aggregation level of the data, which has three aspects: geographical, temporal and institutional level.

The geographical aspect is quite widely applied, which means that institutions are involved from different continents, regions and countries.

Sometimes, the time span is also distinguished in the papers as pre-crisis, crisis and post-crisis analysis. This division is fundamental, thus some methods behave differently in distinct periods. Finally, the level of aggregation is also essential, as the more levels are integrated in the investigation, the deeper can be understood the transmission of risk from lower channels to higher ones.

The third topic, which should be discussed, is the methodology used for the modelling. The majority of the papers did not take into consideration the influence of the market on the risk spillovers because of this was important that Gong et al. (2019) filtered out the impact of the market using the CAPM modell.

Besides, the rarely covered topic is the question of noise, while the efficiency of the methods can depend on the level of innovations (Nucera et al. 2016, Giglio et al. 2016, p.461).

A further common phenomenon is, that the practice use linear models for the analysis, even though non-linear impacts also exist on the market Chao et al. (2015)

Several times, there is a considerable disadvantage of the methods, especially there is no reference point, only historical analysis can confirm that the results show high risk for the financial market or not, like in the case of DY spillover index and DCI. However, the leave-one-out concept was elaborated to show the precision of the DCI approach and compensate for this deficiency.

Lastly, the crucial question is, that ex-post developed risk metrics can be able to serve as real early warning indicators for the turmoils?

Several aspects should be considered as an integral part of the broad analysis.

Methodologically, I am trying to provide a deep, structured analysis with a classic linear approach. I will also specify the framework in order to create a multi-level tool for individual institutions and aggregated data. Nevertheless, I am aiming to return to the basic principle of systemic risk and express the economic loss caused by a systemic event.

Considering heterogeneous market structures, I will focus on large, publicly traded companies from six branches multi-line, life, health, property and casualty, credit and mortgage insurance and reinsurance. I also will reflect on individual and sectoral level linkages and distinguish the periods adapting to the business cycle.

However, I do not treat the question of noisy data and non-linear effects. My research will reflect on existing relations between companies without trying to make forecasts for the future.

IV Data and methodology

IV.1 Methods

Billio et al. (2012) proposed a suitable framework for describing the interconnectedness of financial networks and the importance of individual firms, which was elaborated by Hué et al. (2019) and extended by Song and Taamouti (2019). Subsection IV.1.1 summarizes the original modell, while subsection IV.1.2 the leave-one-out approach and subsection IV.1.5 the extended version of the framework.

IV.1.1 Degree of Granger-causality

Degree of Granger-causality[5] tries to summarize the information incorporated in the pairwise relationship between individual institutions on the level of the system. However, the framework is multiple-use, which means that it can capture the relationship between different aggregates with higher granularity, like subsectors, sectors or whole industries.

I follow the authors' way of thinking in the description of the method (Billio et al. 2012, p.539-541). At the first step, focus on the lowest level of aggregation, on individual institutions.

The starting point of the approach is based on the number of existing relationships between institutions measured by linear Granger-causality (Granger 1969), which highlights not only the existence but also the directionality of linkages.[6] Time series i Granger-causes times series j, if the set of the past information included in i contains information for predicting the values of the time series j. Mathematically saying, characterize two stationary time series R_t^i and R_t^j with α^i, α^j, β^{ij}, β^{ji} coefficients as follows:

$$R_{t+1}^i = \alpha^i R_t^i + \beta^{ij} R_t^j + e_{t+1}^i, \tag{1}$$

$$R_{t+1}^j = \alpha^j R_t^j + \beta^{ji} R_t^j + e_{t+1}^j. \tag{2}$$

Of course, e_{t+1}^i and e_{t+1}^j are white noise processes. Time series i Granger-causes time series j, when β^{ji} is not equal with zero. Similarly, time series j Granger-causes time series i, when β^{ij} significantly differs from zero.[7]

The formal hypothesis test was introduced by Granger (1969) investigating whether the information deriving from one time series helps to predict the evaluation of another one. In the depiction of the test I follow Hué et al. (2019) (Hué et al. 2019, p.91).

[5]The abbreviation of the measure is DCI in the literature, however, Billio et al. (2012) used DGC in their article. Nevertheless, staying consistent with the literature, I use the DCI designation.

[6]Naturally, the applied mathematical formalism is adequate to test non-linear causality, but I will concentrate on linear effects.

[7]The equations were formalized if a single lag is enough to explain the dynamics. In other cases, more lags should be involved.

$$\mathbb{H}_0 : Pr(R_t^i < R | \mathscr{F}_{t-1}) = Pr(R_t^i < R | \mathscr{F}_{t-1}^i), \tag{3}$$

which is true for all R values, and the information sets are designed by (4) and (5). Also, accepting the null hypothesis means that the joint information set does not consist of more information than the information set of the i time series.

$$\mathscr{F}_{t-1} = \left\{(R_s^i, R_s^j)^\top, s \leq t-1\right\}, \tag{4}$$

$$\mathscr{F}_{t-1}^i = \{R_s^i, s \leq t-1\}. \tag{5}$$

The existence of causality is indicated by the (6) formalism.

$$(i \to j) = \begin{cases} 1, \text{ if } j \text{ Granger-causes } i, \\ 0 \text{ otherwise.} \end{cases} \tag{6}$$

Now, I can define DCI, considering that totally $N(N-1)$ relationship exists among institutions, where N denotes the number of market actors. So, the number of existing connections is divided by the number of all possible relationships expressed by (7).

$$DCI = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{i \neq j} (j \to i) \tag{7}$$

However, DCI varies between 0 and 1, but it is not bright what level of the measure expresses growing risk in the financial network. So, the authors defined a K limit, and exceeding K means rising risk in the network (Billio et al. 2012, p.540).

The number of connectedness can be divided according to the direction of edges. The number of incoming edges to one institution is formalised as followings:

$$\#In : (j \to S)|_{DCI \geq K} = \frac{1}{N-1} \sum_{i \neq j} (i \to j)|_{DCI \geq K}, \tag{8}$$

where S expresses the whole system. The outgoing links for a distinct company can be gauged similarly:

$$\#Out : (S \to j)|_{DCI \geq K} = \frac{1}{N-1} \sum_{i \neq j} (j \to i)|_{DCI \geq K}, \tag{9}$$

(8) and (9) can be summarized in one equation, as counting all significant incoming and outgoing.

$$\#In + Out : (j \leftrightarrow S)|_{DCI \geq K} = \frac{1}{2(N-1)} \sum_{i \neq j} (i \to j) + (j \to i)|_{DCI \geq K} \tag{10}$$

(10) expresses the significance of one company, based on the number of existing edges of the Granger-network.

As a further step, consider M different groups of institutions, like banks and insurers and $M = 2$. For the comparison denote λ, $\kappa = 1, ..., M$ indexes. The former measures can be generalized to selected groups, see equations (11), (12) and (13). Equation (11) summarizes the aggregated outgoing causalities, equation (12) catches the total incoming effects. In contrast, equation (13) contains all incoming and outgoing edges from one sector to other.

$$\#In - to - Other : \left((j|\lambda) \to \sum_{\kappa \neq \lambda} (S|\kappa) \right) \Bigg|_{DGC \geq K} = \frac{1}{(M-1)N/M} \sum_{\kappa \neq \lambda} \sum_{i \neq j} \left((j|\lambda) \to (i|\kappa) \right) \Big|_{DGC \geq K} \tag{11}$$

$$\#Out - to - Other : \left(\sum_{\kappa \neq \lambda} (S|\kappa) \to (j|\lambda) \right) \Bigg|_{DGC \geq K} = \frac{1}{(M-1)N/M} \sum_{\kappa \neq \lambda} \sum_{i \neq j} \left((i|\kappa) \to (j|\lambda) \right) \Big|_{DGC \geq K} \tag{12}$$

$$\#In + Out - Other : \left((j|\lambda) \leftrightarrow \sum_{\kappa \neq \lambda} (S|\kappa) \right) \Bigg|_{DGC \geq K} = \frac{\sum_{\kappa \neq \lambda} \sum_{i \neq j} \left((i|\kappa) \to (j|\lambda) + (j|\lambda) \to (i|\kappa) \right) \Big|_{DGC \geq K}}{2(M-1)N/M} \tag{13}$$

As a final remark, I must mention that the DCI method is a linear approach, which is not suitable for expressing the non-linear effects of the financial market. However, the risk spillovers on the financial market contain non-linear effects proved by Chao et al. (2015). For this reason, Billio et al. (2012) also applied non-linear Granger-causality in their paper, but this is not part of the current analysis.

IV.1.2 Leave-one-out approach

To introduce the leave-one-out concept, firstly, I have to characterize a modified form of the Granger-causality in mean (Granger 1980, 1988, Sims 1972, 1980) hypothesis (3).

$$\mathbb{H}_{0,1} : \mathbb{E}(R_t^i|\mathscr{F}_{t-1}) = \mathbb{E}(R_t^i|\mathscr{F}_{t-1}^i) \tag{14}$$

This statement can be tested with the (15) test statistics.

$$U_{j\to i} = T\ln\left(\frac{\hat{\sigma}_{i,2}^2}{\hat{\sigma}_{i,1}^2}\right), \tag{15}$$

where T symbolises the sample size, $\hat{\sigma}_{i,2}^2$ and $\hat{\sigma}_{i,1}^2$ are the estimated variances of the residuals $\hat{\epsilon_{i,1}}$ and $\hat{\epsilon_{i,2}}$ deriving from the (16) and (17) equations.

$$R_t^i = c_1 + \sum_{s=1}^{M} \phi_s R_{t-s}^i + \sum_{s=1}^{M} \gamma_s R_{t-s}^j + \epsilon_{i,1,t}, \tag{16}$$

$$R_t^i = c_2 + \sum_{s=1}^{M} \delta_s R_{t-s}^i + \epsilon_{i,2,t}. \tag{17}$$

Where M lags are included in the modell, c_1 and c_2 are constants, while ϕ_s, γ_s and δ_s are coefficients. Besides, $U_{j\to i}$ test statistics follow an asymptotic chi-squared distribution with M degree of freedom. Denote η the significance level and $1-\eta$ the confidence level, if the $U_{j\to i}$ test statistics is unambiguously greater then the critical value ($\chi_{1-\eta}^2(M)$), than the null hypothesis will be rejected. So, an indicator can be defined as measuring significant linkages among financial companies.

$$\mathbb{I}(U_{j\to i} > \chi_{1-\eta}^2(M) = \begin{cases} 1, \text{ if } j \text{ Granger-causes } i, \\ 0 \text{ otherwise.} \end{cases} \tag{18}$$

Counting directed linkages, Hué et al. (2019) modified metrics $\#In + Out$ based on (10) and (18) equations to indicate the weight of an institution in the network.

$$InOut_k = \frac{1}{2(N-1)} \sum_{\substack{j=1 \\ j\neq k}}^{N} \left[\mathbb{I}(U_{k\to j} > \chi_{1-\eta}^2(M) + \mathbb{I}(U_{j\to k} > \chi_{1-\eta}^2(M)\right], \tag{19}$$

where the (19) equation measures the incoming and outgoing edges.

Hué et al. (2019) also re-formalised the DCI measure as the level of Granger-causality (LGC) see equation (7). Except that the LGC does not normalise the significant number of relationships with the number of all possible linkages.

$$LGC = \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i\neq j}}^{N} (\mathbb{I}(U_{j\to i} > \chi^2_{1-\eta}(M)) \tag{20}$$

Moreover, leaving out the k^{th} institution from the sample, the LGC_{-k} is defined as follows for a smaller $N-1$ large sample:

$$LGC_{-k} = \sum_{i=1}^{N-1} \sum_{\substack{j=1 \\ j\neq i, j\neq k}}^{N-1} \mathbb{I}(U_{j\to i} > \chi^2_{1-\eta}(M)). \tag{21}$$

However, Hué et al. (2019) pointed out that this form of the LGC will neglect the linkages coming from the left-out institution, but indirect causalities will be reserved in the system.

Let illustrate the indirect relationships using an example (Hué et al. 2019, p.92-93). Imagine that there are three institutions on the market A, B and C. A not direct, also "intermediated" effect exists in the system, if A Granger-cause institution B, which has an impact on the company C. When A is eliminated from the system, its indirect influence will be preserved by institution B. This phenomenon expects some modification in the definition of LGC_{-k}.

$$LGC_{-k} = \sum_{i=1, i\neq k}^{N-1} \sum_{\substack{j=1 \\ j\neq k, j\neq i}}^{N-1} \mathbb{I}(U_{j\to i|k} > \chi^2_{1-\eta}(M)), \tag{22}$$

where the $U_{j\to i|k}$ represents the conditional Granger-causality test, this test can detect that the effects are mediated or directly incoming to the institutions (Geweke 1984).

Formally, the conditional Granger-causality is like the (23) equation, more precisely:

$$U_{j\to i|k} = T \ln\left(\frac{\tilde{\sigma}^2_{i,2}}{\tilde{\sigma}^2_{i,1}}\right), \tag{23}$$

where T is the sample size, $\tilde{\sigma}^2_{i,2}$ and $\tilde{\sigma}^2_{i,1}$ are the calculated variances of the residuals $\tilde{\epsilon}_{i,1}$ and $\tilde{\epsilon}_{i,2}$ calculated in the (16) and (17) equations.

$$R^i_t = c_1 + \sum_{s=1}^{M} \phi_s R^i_{t-s} + \sum_{s=1}^{M} \gamma_s R^j_{t-s} + \sum_{s=1}^{M} \psi_s R^k_{t-s} + u_{i,1,t}, \tag{24}$$

$$R_t^i = c_2 + \sum_{s=1}^{M} \delta_s R_{t-s}^i + \sum_{s=1}^{M} \theta_s R_{t-s}^k + u_{i,2,t}. \tag{25}$$

Where ψ_s and θ_s are the new parameters in comparison to (16) and (17) equations.

The LOO measure can be described by the help of LGC (20) and LGC_{-k} (22) for the distinct institution k:

$$\Delta LGC_k = \frac{LGC - LGC_{-k}}{LGC}. \tag{26}$$

This gauge summarizes the systemic importance of financial institutions neglecting implicit effects from the system. Also, ΔLGC_k expresses in percentage, if a company is excluded from the financial network to what extent will change the connectedness between remaining institutions. Technically, the high positive values mean deeply interconnected institutions in the network, which can pose systemic risk, while companies with low values are secure. Thus the bankruptcy of central institutions will dramatically raise the interconnectedness in the network. In contrast, the drop-out of less important firms will reduce interconnectedness.

Compared to the approach of Billio et al. (2012), the LOO was found to be more compatible, identifying the G-SIIs published by the Financial Stability Board (Hué et al. 2019, p.102). Although the ranking of SIIs reported by the Financial Stability Board is considered as a reference point in the literature, the relevance of the mentioned ranking is not clear.

IV.1.3 Holm-Bonferroni correction

The multiple hypothesis testing problems arise when pairwise Granger-causality tests are performed, which means that the higher the number of tests, the higher the probability of rejecting the null hypothesis, which can result spurious consequences deriving from the data. This problem is solved by the family-wise error rates (FWER) approach for multiple hypothesis tests. Several methods belong to this framework, I have chosen the widely spread Holm-Bonferroni Correction (also called Holm's Sequential Bonferroni Procedure (Holm 1979)).

The steps of the correction are the followings:

1. Consider $p_1, p_2, ...p_n$ p-values deriving from the hypothesis tests.
2. Rank the p-values in ascending order: $p_1^* \leq p_2^* \leq ... \leq p_n^*$
3. Calculate modified p-values. Select the targeted level of significance (α). Transform the significance level for the ordered p-values using $\alpha_{rank}^* = \frac{\alpha}{n-rank+1}$, where $rank = 1,..,n$ and signs position in the ordered sample. So the lowest p-value has the rank 1, and the highest n.
4. Compare p_{rank}^* and α_{rank}^* ($rank = 1,..,n$) given by (27).

$$\begin{cases} p^*_{rank} < \alpha^*_{rank} & \text{Reject the null hypothesis} \\ p^*_{rank} \geq \alpha^*_{rank} & \text{Cannot reject the null hypothesis} \end{cases} \tag{27}$$

IV.1.4 Criticism of the ΔLGC_k approach

In this section, I will show that the leave-one-out approach and the ΔLGC_k are not carefully defined, which can cause distortions in empirical use. Nevertheless, the main idea of the framework proposed by Hué et al. (2019) should not be rejected, while the basic concept is correct. Primarily, the Granger-causality must be based on the knowledge of the true network, which means the real existing relationships between institutions represented by nodes. If the real network can be extracted from the observations, then the results of the article published by Hué et al. (2019) will be valid.

Firstly, I explain why ΔLGC_k is inappropriate for detecting SIFI-s. After that, I make some remarks to its use and deficiencies for representing systemic risk for the whole network.

The ΔLGC_k approach was introduced to improve the $InOut_k$ measure counting the proportion of incoming and outgoing edges from the k^{th} node. The innovation was necessary, while the systemic risk measures usually ignore the question of spurious edges arising in the network. However, the existence of indirect linkages can induce the misclassification of SIFIs, which questions the economic consequences drawn from the false risky institution rankings. So, it is a fundamental question to treat the spurious effects of the financial networks to present reliable analysis about the stability of the system.

Hué et al. (2019) proposed a framework to filter the false connections from the networks. In this section, I revise the ΔLGC_k measure, I will express that it cancels only some distinct spurious edges, and the interpretability is hard thanks to the theoretical domain of the measure.

Hué et al. (2019) emphasized that the $InOut_k$ measure is only appropriate for networks without spurious connections, but the real networks usually contain false linkages, which can cause distortions. To understand the authors' motivation, consider the example of Hué et al. (2019) firstly. The authors described a graph, which is a proper demonstration of filtering out all spurious connections. As figure 1 demonstrate a small network with three institutions. There are existing real connections from institution 3 to institution 1 and institution 2 indicated by black lines, while a spurious relationship between institution 1 and institution 2 described by a waved grey arrow.

However, leaving-out the institutions individually, the following relationships can be observed. One directed edge from institution 3 to institution 1, and other one between institution 3 and institution 2. Institution 1 and institution 2 are not connected. The described connection map means that one spurious edge is filtered from the network, and the real structure of the graph can be reconstructed.

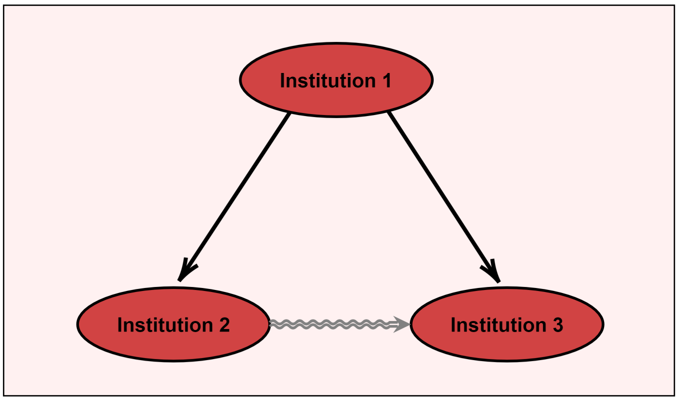


Figure 1: Example I. of Hué et al. (2019) for spurious edges in financial networks

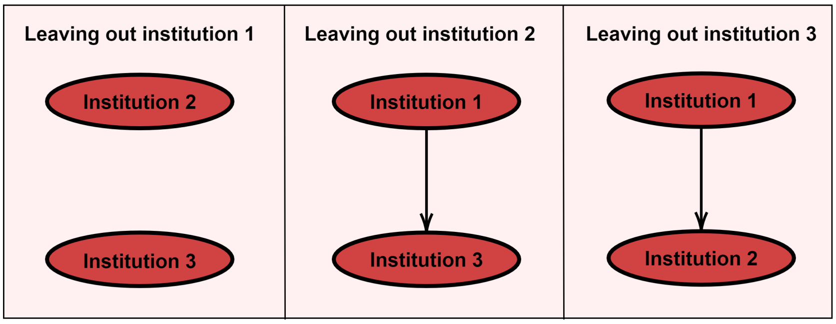


Figure 2: Example I. of Hué et al. (2019), when leave-one-out approach clears spurious edges from the network

The solution is quite simple and attractive, and a non-mentioned advantage of the approach is that in particular circumstances, the real network (without false linkages) can be reproduced, putting together the edges. An example of this is presented in figure 2.

Nevertheless, this framework has methodological and economic deficiencies. On the one hand, the weights can include spurious effects. At the same time, the methodology does not filter all spurious effects, which implies that the cited example of Hué et al. (2019) is only a special case, not the general rule (figure 1 and figure 2.)

On the other hand, the weights created by the ΔLGC_k are not reflected in the $InOut_k$ values of the real network.

As the first step, I show a network, which demonstrates that not all false edges disappear using the conditional Granger-causality test. So, consider the modified version of the first example displayed in figure 3. Two new institutions are added to the network (figure 1): institution 4 and institution 5. Also, new connections appear between institution 2 and institution 5 and a spurious connection between institution 4 and institution 5.

Apply the leave-one-out framework to this modified graph. Figure 2 displays the remaining edges dropping the k^{th} node ($k = 1,2,3,4,5$) from the network. Spurious edges are included in three cases in figure 2. When institution 3, institution 1 and institution 5 are excluded, which means that the conditional Granger-test clears only the adjacent spurious edges, not all. Furthermore, this means that putting together the leave-one-out networks (figure 2) does not result the original connectedness graph without spurious edges. So, only using this correction method does not grant an adequate picture of the real network and real connectedness structure.

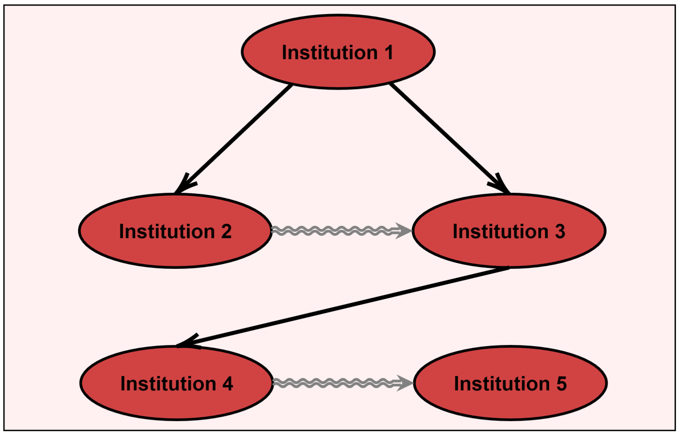


Figure 3: Modified example I. of Hué et al. (2019)

Furthermore, the ΔLGC_k is poorly defined, which induces inappropriate economic interpretability and false conclusions drawing from the results.

A good starting point is the definition of the measure: $\Delta LGC_k = \frac{LGC-LGC_{-k}}{LGC}$. To conclude the results for policymakers, the range of the gauge should be known, but the authors' do not clear this topic so that I will distinguish two fundamentally different cases. (Excluding the point, when $LGC = LGC_{-k}$, which results that the ΔLGC_k becomes zero.)

Condition	Max value	Max argument	Min value	Min argument
$LGC > LGC_{-k}$	1	$LGC_{-k} = 0$	$\frac{1}{(n-1)(n-2)+1} \searrow 0$	$LGC_{-k} = (n-1)(n-2)$ $LGC = (n-1)(n-2)+1$
$LGC < LGC_{-k}$	$\frac{1}{(n-1)(n-2)-1} \nearrow 0$	$LGC_{-k} = (n-1)(n-2)$ $LGC = (n-1)(n-2)-1$	$1-(n-1)(n-2)$	$LGC_{-k} = (n-1)(n-2)$ $LGC = 1$

Table 9: The domain of ΔLGC_k

The table shows that $\Delta LGC_k \in [1-(n-1)(n-2), 1]$ asymptotically $[-\infty, 1]$, as n converges to infinity. The domain of the measure questioned the interpretability of the approach, while the authors described SIFIs owing ΔLGC_k near to one, and less vulnerable companies possess lower values.However, the authors miss highlighting that ΔLGC_k can fall below zero,

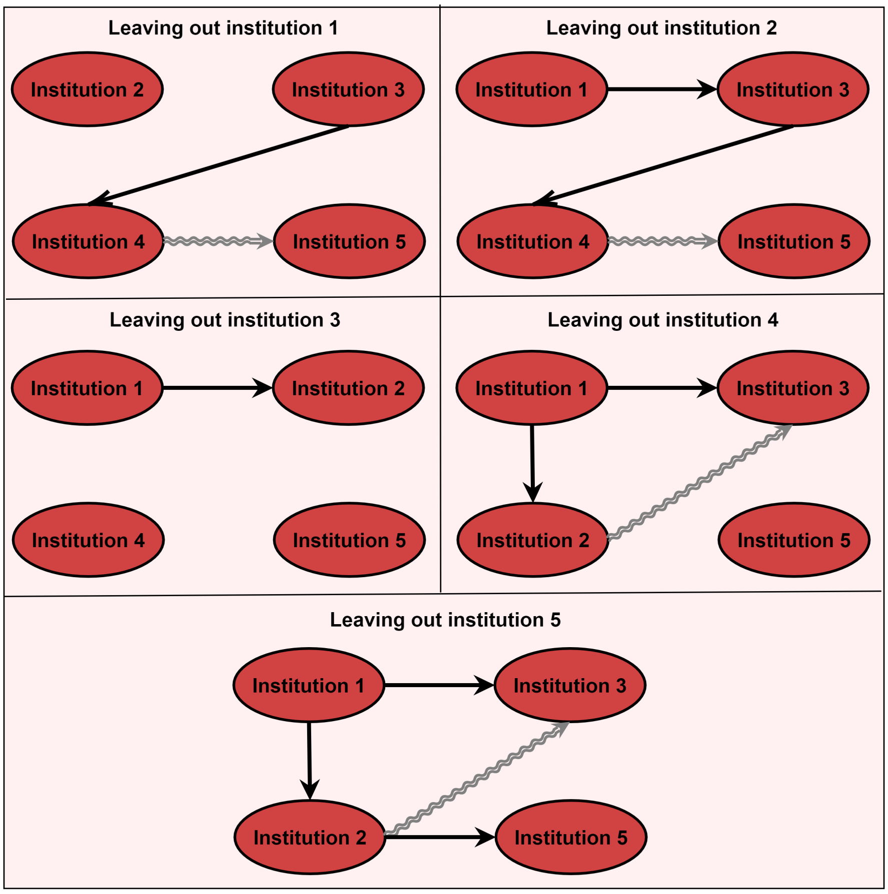


Figure 4: Modified example I. of Hué et al. (2019), when leave-one-out approach does not clear all spurious edges from the network

and the measure has no lower bounds asymptotically. However, at least on finite samples, it is not symmetric, which aggravates the comparison of secure and vulnerable companies.

Figure 5 presents an example of how ΔLGC_k values can variate if one institution is dropped out of the network. The y-axis represents the connections in the starting network, which can be maximum $n(n-1)$ if n institutions are included, while x-axis describes the possible linkages after leaving out 1 company. An important remark is that in the starting network, I assume that there is at least 1 connection, else ΔLGC_k is not meaningful. Nevertheless, in the modified network zero connection is allowed.

Understanding the ΔLGC_k gauge needs to overthink the two cases indicated in the table 9.

When $LGC > LGC_{-k}$, then removing the k^{th} institution from the system will delete some connections, which indicates that an embedded firm is in the network. Figures 6 and 7 presents an example for this case. Leaving-out institution 2 will cancel the spurious edge between institution 1 and institution 3.

	0	1	2	3	4	5	6	7	8	9	10	11	12
1	1,00	0,00	-1,00	-2,00	-3,00	-4,00	-5,00	-6,00	-7,00	-8,00	-9,00	-10,00	-11,00
2	1,00	0,50	0,00	-0,50	-1,00	-1,50	-2,00	-2,50	-3,00	-3,50	-4,00	-4,50	-5,00
3	1,00	0,67	0,33	0,00	-0,33	-0,67	-1,00	-1,33	-1,67	-2,00	-2,33	-2,67	-3,00
4	1,00	0,75	0,50	0,25	0,00	-0,25	-0,50	-0,75	-1,00	-1,25	-1,50	-1,75	-2,00
5	1,00	0,80	0,60	0,40	0,20	0,00	-0,20	-0,40	-0,60	-0,80	-1,00	-1,20	-1,40
6	1,00	0,83	0,67	0,50	0,33	0,17	0,00	-0,17	-0,33	-0,50	-0,67	-0,83	-1,00
7	1,00	0,86	0,71	0,57	0,43	0,29	0,14	0,00	-0,14	-0,29	-0,43	-0,57	-0,71
8	1,00	0,88	0,75	0,63	0,50	0,38	0,25	0,13	0,00	-0,13	-0,25	-0,38	-0,50
9	1,00	0,89	0,78	0,67	0,56	0,44	0,33	0,22	0,11	0,00	-0,11	-0,22	-0,33
10	1,00	0,90	0,80	0,70	0,60	0,50	0,40	0,30	0,20	0,10	0,00	-0,10	-0,20
11	1,00	0,91	0,82	0,73	0,64	0,55	0,45	0,36	0,27	0,18	0,09	0,00	-0,09
12	1,00	0,92	0,83	0,75	0,67	0,58	0,50	0,42	0,33	0,25	0,17	0,08	0,00
13	1,00	0,92	0,85	0,77	0,69	0,62	0,54	0,46	0,38	0,31	0,23	0,15	0,08
14	1,00	0,93	0,86	0,79	0,71	0,64	0,57	0,50	0,43	0,36	0,29	0,21	0,14
15	1,00	0,93	0,87	0,80	0,73	0,67	0,60	0,53	0,47	0,40	0,33	0,27	0,20
16	1,00	0,94	0,88	0,81	0,75	0,69	0,63	0,56	0,50	0,44	0,38	0,31	0,25
17	1,00	0,94	0,88	0,82	0,76	0,71	0,65	0,59	0,53	0,47	0,41	0,35	0,29
18	1,00	0,94	0,89	0,83	0,78	0,72	0,67	0,61	0,56	0,50	0,44	0,39	0,33
19	1,00	0,95	0,89	0,84	0,79	0,74	0,68	0,63	0,58	0,53	0,47	0,42	0,37
20	1,00	0,95	0,90	0,85	0,80	0,75	0,70	0,65	0,60	0,55	0,50	0,45	0,40

Connections remaining in the network after leaving-out one institution

Connections in the initial network

Figure 5: All possible ΔLGC_k values, if $n = 5$ company is in the initial network

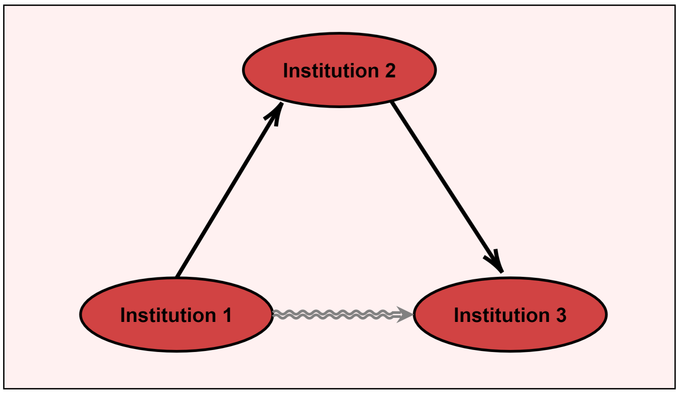


Figure 6: Example II. of Hué et al. (2019)

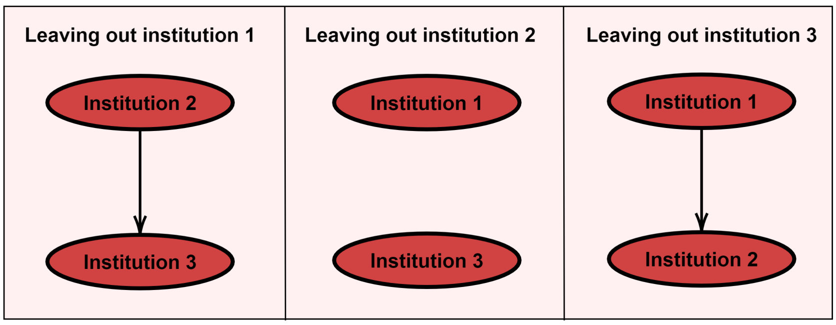


Figure 7: Leave-one-out relations of example II. (Hué et al. 2019)

While in the other case ($LGC < LGC_{-k}$), excluding a node from the network will create new connections, which means that the selected institution absorbs risk, which cannot spill over to the others. This case seems to be uncertain, but in some cases can happen, see the simplified example of Song and Taamouti (2019) (Song and Taamouti 2019, p.916). I plotted the network graph of the mentioned example in figure 8 and the leave-one-out analysis on figure 9.

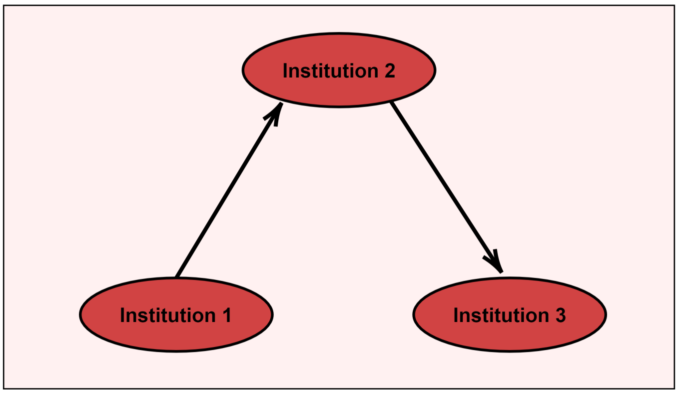


Figure 8: Network graph modified example of Song and Taamouti (2019)

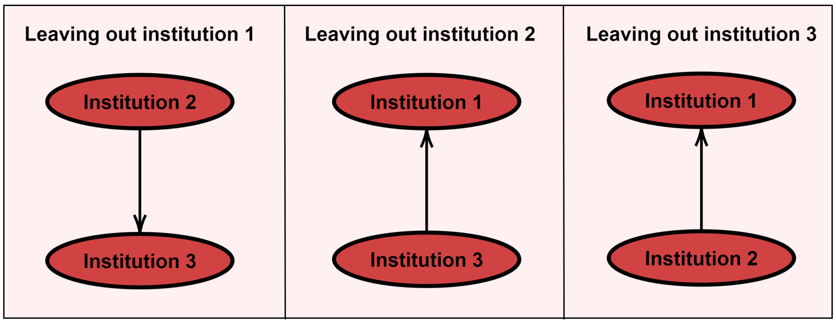


Figure 9: Leave-one-out relations of the modified example of Song and Taamouti (2019)

Having considered the domain of ΔLGC_k shed light on the confusing mathematical properties and weak economic interpretability of the measure, which requires a revision for empirical analysis. A possible extension can be specified following the idea of Song and Taamouti (2019), and I will present in the next section.

IV.1.5 Testing indirect and spurious effects

Song and Taamouti (2019) formalised the definitions of indirect and spurious causalities and the characterization of the testing procedure. I present their approach (Song and Taamouti 2019, p.914-915).

Let consider three time series $\{X_t, t \in \mathbb{Z}\}$, $\{Y_t, t \in \mathbb{Z}\}$ and $\{Z_t, t \in \mathbb{Z}\}$, while $I_X(t) = \{X(s) : s \leq t\}$, $I_Y(t) = \{Y(s) : s \leq t\}$ and $I_Z(t) = \{Z(s) : s \leq t\}$ the information sets of X, Y and Z stochastic processes. Moreover, $I(t) = I_X(t) \cup I_Y(t) \cup I_Z(t)$ denotes the combined information set for a given time t. Similarly can define $I_{-X}(t) = I_Y(t) \cup I_Z(t)$, $I_{-Y}(t) = I_X(t) \cup I_Z(t)$ and $I_{-Z}(t) = I_X(t) \cup I_Y(t)$.

The definition of indirect causality is the following (Song and Taamouti 2019, p.914):
Y indirectly Granger-causes X, if:

1. Y Granger-causes X with respect to the information set $I_X(t)$:

$$P\Big(X_{t+1}|I_X(t)\Big) \neq P\Big(X_{t+1}|I_{-Z}(t)\Big), \text{ for some } t > \omega. \tag{28}$$

2. Y does not Granger-cause X with respect to the information set I_{-Y}:

$$P\Big(X_{t+1}|I_{-Y}(t)\Big) = P\Big(X_{t+1}|I_{-Z}(t)\Big), \forall t > \omega. \tag{29}$$

3. Y Granger-causes Z, while Z Granger-causes X with respect to the information sets $I_{-Y}(t)$ and $I_{-Z}(t)$:

$$\begin{aligned} &P\Big(Z_{t+1}|I_{-Y}(t)\Big) \neq P\Big(Z_{t+1}|I(t)\Big), \text{ for some } t > \omega \\ &P\Big(X_{t+1}|I_{-Z}(t)\Big) \neq P\Big(X_{t+1}|I(t)\Big), \text{ for some } t > \omega, \end{aligned} \tag{30}$$

   where ω is the "starting date" of the observed sample.

All items are needed to fulfill the mentioned requirements in order to call Y as an indirect Granger-cause of X. A proper example for indirect causality is the example II. from Hué et al. (2019) (see figure 19), while indirect causality tries to express that the node transfers risk from one node to other.

Definition of spurious causality (Song and Taamouti 2019, p.914-915). Y spuriously Granger-causes X if,

1. Type I.

   (a) Y Granger-causes X with respect to the information set $I_{-Y}(t)$:

$$P\Big(X_{t+1}|I_{-Y}(t)\Big) \neq P\Big(X_{t+1}|I(t)\Big), \text{ for some } t > \omega. \tag{31}$$

   (b) Y does not Granger-causes X with respect to the information set $I_X(t)$:

$$P\Big(X_{t+1}|I_{-Y}(t)\Big) \neq P\Big(X_{t+1}|I(t)\Big), \forall t > \omega. \tag{32}$$

(c) Y Granger-causes Z, Z Granger-causes X with respect to the information sets $I_{-Y}(t)$ and $I_{-Z}(t)$:

$$\begin{aligned} P\Big(Z_{t+1}|I_{-Y}(t)\Big) &\neq P\Big(Z_{t+1}|I(t)\Big), \text{ for some } t > \omega, \\ P\Big(X_{t+1}|I_{-Z}(t)\Big) &\neq P\Big(X_{t+1}|I(t)\Big), \text{ for some } t > \omega. \end{aligned} \tag{33}$$

Type I. spurious causality can be illustrated with the network 8. At the same time, the definition induces the phenomenon, that including an institution in the network, it can reduce the number of connections via absorbing risk. If this institution is dropped out, new linkages appear. This type of causality was neglected by Hué et al. (2019).

2. Type II.

(a) Y Granger-causes X with respect to the information set $I_X(t)$:

$$P\Big(X_{t+1}|I_X(t)\Big) \neq P\Big(X_{t+1}|I_{-Z}(t)\Big), \text{ for some } t > \omega. \tag{34}$$

(b) Y does not Granger-causes X with respect to the information set $I_{-Y}(t)$:

$$P\Big(X_{t+1}|I_{-Y}(t)\Big) \neq P\Big(X_{t+1}|I(t)\Big), \forall t > \omega. \tag{35}$$

(c) Z Granger-causes Y, while Z Granger-causes X with respect to the information set $I_{-Z}(t)$:

$$\begin{aligned} P\Big(Y_{t+1}|I_{-Z}(t)\Big) &\neq P\Big(Y_{t+1}|I(t)\Big), \text{ for some } t > \omega, \\ P\Big(X_{t+1}|I_{-Z}(t)\Big) &\neq P\Big(X_{t+1}|I(t)\Big), \text{ for some } t > \omega. \end{aligned} \tag{36}$$

The Type II. spurious connection is visualized on network plot 2, which illustrates a situation when an external effect causes the connection between unconnected nodes.

The authors pointed out that the X and Y are observable variables, while Z a latent one. So, you can see that the framework of Song and Taamouti (2019) is an extension of the method proposed by Hué et al. (2019) if Z is appropriately chosen.

The extension based on the fact that Z is the first principal component of the available data set. Denote $W = (w_1, ..., w_T)^T$ a $T \times N$ multi-dimensional time series, Λ the loading matrix, f_t the factors, which decompose the original observations as follows:

$$w_t = \Lambda f_t + \widetilde{\epsilon}_t \tag{37}$$

Formally, the first principal component contains the most information about the data, and it will represent the other time series in the analysis.

Firstly, for testing the definition 1 (indirect causalities), the following procedure should be applied (Song and Taamouti 2019, p.918-920).

1. Testing the first condition of definition 1. Consider the equation (38), a two dimensional VAR modell.

$$X_{t+1} = \mu + \sum_{i=1}^{p} \beta_i X_{t+1-i} + \sum_{j=1}^{q} \alpha_j Y_{t-1-j} + \epsilon_{t+1} \tag{38}$$

   Apply an F-test to the coefficients of Y_t according to (39) hypothesis.

$$\begin{aligned} &H_0 : \alpha_1 = \ldots = \alpha_q = 0, \\ &H_1 : \exists \text{ at least one } \alpha_k \neq 0, k = 1,.., q. \end{aligned} \tag{39}$$

   If the null hypothesis is rejected, then test condition 2. If not, you can conclude that Y directly Granger-causes X.

2. Add as a new explaining variable the first principal component and apply a Wald-test on the lambda coefficients based on (41).

$$X_{t+1} = \eta + \sum_{i=1}^{\overline{p}} \gamma_i X_{t+1-i} + \sum_{j=1}^{\overline{q}} \lambda_j Y_{t-1-j} + \sum_{l=1}^{\overline{h}} \theta_l f_{t-1-l} + e_{t+1} \tag{40}$$

$$\begin{aligned} &\overline{H}_0 : \lambda_1 = \ldots = \lambda_{\overline{q}} = 0, \\ &\overline{H}_1 : \exists \text{ at least one } \lambda_k \neq 0, k = 1,.., \overline{q}. \end{aligned} \tag{41}$$

   If $\overline{H}$ is not rejected, then consider condition 3. Otherwise, you observed that Y directly Granger-causes X.

3. As the last step, check the third condition.

$$\begin{aligned} f_{t+1} &= \nu + \sum_{i=1}^{\dot{p}} \kappa_i X_{t+1-i} + \sum_{j=1}^{\dot{q}} \psi_j Y_{t-1-j} + \sum_{l=1}^{\dot{h}} \rho_l f_{t-1-l} + u_{t+1}, \\ X_{t+1} &= \overline{\omega} + \sum_{i=1}^{\ddot{p}} \xi_i X_{t+1-i} + \sum_{j=1}^{\ddot{q}} \delta_j Y_{t-1-j} + \sum_{l=1}^{\ddot{h}} \zeta_l f_{t-1-l} + \varepsilon_{t+1} \end{aligned} \tag{42}$$

   Two null hypotheses should be rejected in (43) to conclude that between Y and X is only an indirect relationship.

$$\begin{aligned}&\dot{H}_0 : \psi_1 = ... = \psi_{\dot{q}} = 0,\\&\dot{H}_1 : \exists \text{ at least one } \psi_k \neq 0, k = 1,..,\dot{q},\\&\ddot{H}_0 : \zeta_1 = ... = \zeta_{\ddot{h}} = 0,\\&\ddot{H}_1 : \exists \text{ at least one } \zeta_r \neq 0, r = 1,..,\ddot{h}.\end{aligned} \tag{43}$$

Not only indirect, but also spurious relations can distort the overview of the real network, so it is necessary to check for spurious connections. The procedure checks the three conditions of Type 1 definition.

1. Test the first condition of definition 1.

$$X_{t+1} = \mu + \sum_{i=1}^{p} \beta_i X_{t+1-i} + \sum_{j=1}^{q} \alpha_j Y_{t-1-j} + \sum_{l}^{k} \pi_l f_{t+1-l} + \epsilon_{t+1} \tag{44}$$

   The coefficients of Y_t are tested based on the (45) hypothesis.

$$\begin{aligned}&H_0^{(1)} : \alpha_1 = ... = \alpha_q = 0,\\&H_1^{(1)} : \exists \text{ at least one } \alpha_k \neq 0, k = 1,..,q.\end{aligned} \tag{45}$$

   If the null hypothesis is rejected, to detect spurious effects check condition 2. In other cases, Y directly Granger-causes X.

2. Add as a new explaining variable the first principal component and apply a Wald-test on the lambda coefficients based on (47).

$$X_{t+1} = \eta + \sum_{i=1}^{\overline{p}} \beta_i X_{t+1-i} + \sum_{j=1}^{\overline{q}} \alpha_j Y_{t-1-j} + e_{t+1} \tag{46}$$

$$\begin{aligned}&\overline{H}_0 : \alpha_1 = ... = \alpha_{\overline{q}} = 0,\\&\overline{H}_1 : \exists \text{ at least one } \alpha_k \neq 0, k = 1,..,\overline{q}.\end{aligned} \tag{47}$$

   If the null hypothesis is not rejected, then test condition 3. Otherwise, Y Granger-causes X.

3. Last but not least, a joint Wald-test will be applied in step (48).

$$\begin{aligned}f_{t+1} &= \nu + \sum_{i=1}^{\dot{p}} \kappa_i X_{t+1-i} + \sum_{j=1}^{\dot{q}} \psi_j Y_{t-1-j} + \sum_{l=1}^{\dot{h}} \rho_l f_{t-1-l} + u_{t+1},\\X_{t+1} &= \overline{\omega} + \sum_{i=1}^{\ddot{p}} \xi_i X_{t+1-i} + \sum_{j=1}^{\ddot{q}} \delta_j Y_{t-1-j} + \sum_{l=1}^{\ddot{h}} \zeta_l f_{t-1-l} + \varepsilon_{t+1}\end{aligned} \tag{48}$$

Both null hypotheses must be rejected in order to detect spurious connections between Y and X.

$$\begin{aligned}
&\dot{H}_0^{(1)} : \psi_1 = ... = \psi_{\dot{q}} = 0,\\
&\dot{H}_1^{(1)} : \exists \text{ at least one } \psi_k \neq 0, k = 1,..,\dot{q},\\
&\ddot{H}_0^{(1)} : \zeta_1 = ... = \zeta_{\ddot{h}} = 0,\\
&\ddot{H}_1^{(1)} : \exists \text{ at least one } \zeta_r \neq 0, r = 1,..,\ddot{h}.
\end{aligned} \tag{49}$$

Finally, definition 2 of spurious causality will be tested.

1. Similarly to check definition 1, the coefficients of Y values will be controlled.

$$X_{t+1} = \mu + \sum_{i=1}^{p} \beta_i X_{t+1-i} + \sum_{j=1}^{q} \alpha_j Y_{t-1-j} + \epsilon_{t+1}. \tag{50}$$

   To investigate the Granger-causality, the hypothesis (51) will be verified.

$$\begin{aligned}
&H_0^{(2)} : \alpha_1 = ... = \alpha_q = 0,\\
&H_1^{(2)} : \exists \text{ at least one } \alpha_k \neq 0, k = 1,..,q.
\end{aligned} \tag{51}$$

   Rejecting the null hypothesis, consider the next condition. In other cases, the conclusion is that Y Granger-causes X.

2. Extending the VAR modell with the factor, repeatedly, a Wald-test will be applied to lambda coefficients in equation (53).

$$X_{t+1} = \eta + \sum_{i=1}^{\overline{p}} \gamma_i X_{t+1-i} + \sum_{j=1}^{\overline{q}} \lambda_j Y_{t-1-j} + \sum_{l=1}^{\overline{h}} \theta_l f_{t+1-l} + e_{t+1} \tag{52}$$

$$\begin{aligned}
&\overline{H}_0^{(2)} : \alpha_1 = ... = \alpha_{\overline{q}} = 0,\\
&\overline{H}_1^{(2)} : \exists \text{ at least one } \alpha_k \neq 0, k = 1,..,\overline{q}.
\end{aligned} \tag{53}$$

   If the null hypothesis is not rejected, then the following condition should be proved to confirm spurious effects, else Y Granger-causes X.

3. A final F-test will be applied in (54) to identify spurious effects.

$$
\begin{aligned}
f_{t+1} &= \nu + \sum_{i=1}^{\dot{p}} \kappa_i X_{t+1-i} + \sum_{j=1}^{\dot{q}} \psi_j Y_{t-1-j} + \sum_{l=1}^{\dot{h}} \rho_l f_{t-1-l} + u_{t+1}, \\
X_{t+1} &= \overline{\omega} + \sum_{i=1}^{\ddot{p}} \xi_i X_{t+1-i} + \sum_{j=1}^{\ddot{q}} \delta_j Y_{t-1-j} + \sum_{l=1}^{\ddot{h}} \zeta_l f_{t-1-l} + \varepsilon_{t+1}
\end{aligned}
\tag{54}
$$

The two hypotheses should be rejected at the same time for confirming spurious linkages.

$$
\begin{aligned}
&\dot{H}_0^{(1)} : \rho_1 = \ldots = \rho_{\dot{q}} = 0, \\
&\dot{H}_1^{(1)} : \exists \text{ at least one } \rho_k \neq 0, k = 1, .., \dot{q}, \\
&\ddot{H}_0^{(1)} : \zeta_1 = \ldots = \zeta_{\ddot{h}} = 0, \\
&\ddot{H}_1^{(1)} : \exists \text{ at least one } \zeta_r \neq 0, r = 1, .., \ddot{h}.
\end{aligned}
\tag{55}
$$

The formal way to calculate test statistics can be found in the Appendix section C.1.

IV.1.6 Validation methods for vulnerability rankings

Selecting the most suitable framework for the identification of systemically important financial institutions is a complicated field, while several approaches exist, which are potential tools for the detection of vulnerable parts of the networks. I raise only some standard validation methods to highlight the possible ways of testing the results.

Two main ways exist for checking the reliability of different frameworks: modell building and comparison with the published list of global systemically important institutions. The list is published annually by the Financial Stability Board (Financial Stability Board 2019) for banks (G-SIB), and by the Financial Stability Board and the International Association of Insurance Supervisors (Financial Stability Board 2016) for insurers (G-SII).

The modell building approach applies, in general, the value of the measures as independent variables in regressions (Grundke and Tuchscherer 2019, Irresberger et al. 2017, Zhang et al. 2015) or panel modells (Chen et al. 2013, Hué et al. 2019) in order to test the explaining power of the potential early-warning indicators. However, sometimes the rank number is also involved in the analysis (Abendschein and Grundke 2018), while Berdin and Sottocornola (2015) applied difference-in-difference analysis for robustness check.

The more complicated question is what should be chosen as the dependent variable. For forecasting purposes, Grundke and Tuchscherer (2019) used logit modell to test that the bankruptcy of an institution will cause the fall of a given proportion of market players. At the same time, Hué et al. (2019) predicted the possible future loss based on the leave-one-out framework. The spillover risk, represented by realized covariance, was measured by Zhang et al. (2015). The authors also explained by-and-hold-return expressing systemic risk introduced by Acharya et al. (2010) and capital shortfall, too.

The more convenient way is the comparison of the results with the published systemically important institutions. Although, you cannot be sure that the list contains all unstable firms, and the involved companies are really posing high systemic risk. A further deficiency is, that the list of the vulnerable global insurers is available only until 2016.

All in all, the rankings of vulnerable institutions can serve as a help for fast decision making for business companies, credit rating companies, but the more profound understanding assumes econometric analysis to be sure in the outcome of the research.

IV.2 Data

The analysis aims to give a broad overview of the risk of individual insurers, the insurance sector, and the relationship between insurance and banking branches. So, I tried to include as much publicly traded insurers as possible, reflecting on North American and European locations. However, I choose only a selected group of banks to represent the banking sector, while this industry is deeply analysed, so I only want to contrast it to the insurance sector. I selected large and small banks measured by market capitalization from both locations, while I tried not to miss small, but embedded banks, which can pose systemic risk pointed out byLin et al. (2018).

I downloaded 157 insurers and 55 banks data from Bloomberg 28.12.2001 – 31.12.2019. After that, I selected the institutions owing enough information to carry out the analyses. (The selected institutions are listed in the Appendix A.) I split the time horizon to three-part: pre-crises, crises and post-crisis periods to investigate the vulnerability of institutions and the system in a different part of the business cycle. The grouping of periods was based on the selection of Hué et al. (2019). The pre-crisis period started on 2001.12.28 and ended on 2007.06.29, the crises period dated between 2007.07.02 – 2009.06.30 and the post-crisis times from 2009.07.01 to 2019.12.31. Totally 1436, 522 and 2740 days data are included in the distinct periods. The number of institutions at different times are listed in table **??**.

Period	EU insurers	North American insurers	EU banks	North American banks	$\sum$
Pre-crisis	15	62	19	17	113
Crisis	28	77	26	23	154
Post-crisis	30	81	27	23	161

Table 10: Insurers and banks from different locations in the dataset

In order to represent the insurance sector, I grouped companies based on the Industry Classification Benchmark (ICB) codes and the Standard Industrial Classification system. Both classifications are necessary while only using the joint grouping frameworks present the full separation of the insurance industry. Based on the ICB codes I differentiated P/C insurers with subsector code 8536, life insurers with subsector code 8575, financial insurers[8], reinsurers with subsector code 6538, full line insurers with subsector code 8532[9] and banks from subsectors 8355 and 8771. The accident and health insurers are categorized based on the ICB to the life insurance sector, so I refined the classification with SIC codes using 6321 subsector index.

I downloaded daily open, low, high and close prices to calculate inputs for further analysis. In general, not the return is believed to contain the most information about the market, but volatility[10] is a better source of information (Giglio et al. 2016), and CDS data have the most outstanding quality (Rodríguez-Moreno and Peña 2013). Practically, volatility data are available for a more extended period, so that I will use it for the investigation. I calculated volatility based on the methodology proposed by Garman and Klass (2016). Also, the daily volatility for the i^{th} time series is the following:

$$\begin{aligned}\tilde{\sigma}_{it} = 0.511(H_{it} - L_{it})^2 - 0.019[(C_{it} - O_{it})(H_{it} + L_{it} - 2O_{it}) \\ -2(H_{it} - O_{it})(L_{it} - O_{it})] - 0.383(C_{it} - O_{it})^2,\end{aligned} \tag{56}$$

where O_{it}, L_{it}, H_{it} and C_{it} symbolize the natural logarithms of the open, low, high and close prices on day t.

[8]Called financial guarantee and mortgage insurers in ICB framework with subsector codes 8536 and 8779, while surety and title insurers based on SIC framework with 6351 and 6361 subsector codes. I use the name of the SIC framework. So, I name full line insurers operating in several insurance subsectors, while there are also called multi-line insurers in the literature.

[9]Only E-L Financial Corp Ltd operates in subsector 3767 according to the ICB codes, but I have checked the operating area on Bloomberg, which describes well the full line insurance business.

[10]Remark: I applied VAR modells during the analysis in accordance with the empirical literature in order to produce comparable results. Nevertheless, I am aware of the fact that there are special econometric modells for volatility, like the widely used HAR framework proposed by Corsi and Reno (2009).

V Empirical analysis

This part of the paper provides an overview of the European and North American insurance sector and its connections with the banking industry.

V.1 Research design

I utilize the idea of Hué et al. (2019) filtering out non-real edges from the network, extended by Song and Taamouti (2019) applying principal component analysis instead in the VAR modells. To be sure to provide a more precise solution, I implemented a Monte Carlo simulation, which you can see in Appendix C.2. The conclusion was for large samples that the approach of Song and Taamouti (2019) worked well in those cases if the principal component analysis included only the variables, which were initially not represented in the Granger-causality test. Also, the PCA method was not applied to the whole dataset as presented by Song and Taamouti (2019), but only on the other variables to avoid multicollinearity effects. I also made sure that every combination of the data is proper for principal component analysis in all periods. Thus the Kaiser-Meyer-Olkin measure never fell below 0.97, which expresses high adequacy.

The analysis is divided into more dimensions: different periods in the business cycle and aggregation level. I consider pre-crisis, crisis and post-crisis periods, static results and institutional, sectoral and industrial levels of aggregation.

I calculate $InOut_k$, DCI, MES, $SRISK$ and $\Delta CoVaR$. For the quantification, I used the common-sense parameters in the literature, like capital adequacy ratio 8%, market index downturn $C = 40\%$ and the ordinary 5% significance level. For the $\Delta CoVaR$ calculation, I used the state variables of Adrian and Brunnermeier (2011) (see in Appendix C.4), but I have changed market-specific variables to $SP500$ Insurance Index ($S5INSU$) and the STOXX Europe 600 Insurance Index (SXIP) to represent the insurance market on both locations. For the DCI specification, I set the limit (K) to 0.06.

In the different periods, 1436, 522 and 2740 trading days are included and total 14 groups from the insurance and banking industry.

In the following part, I analyse all levels of aggregation in different periods in order to contrast similarities and differences.

V.2 Identification of SIFIs

The first aim of the static analysis is to characterize the systemically important institutions, which can potentially pose systemic risk. So, I calculated several measures to provide an overview of the vulnerability institutions. $InOut$[11] is the measure I mostly rely, while

[11] I introduced $InOut_k$ measure, but this section I write only $InOut$, which means I simply count the incoming and outgoing edges from a node without normalization with the possible relations.

it was calculated based on the true network, while ΔCoVaR, MES, SRISK not.[12] Basu et al. (2017) also emphasised this deficiency of ΔCoVaR.

However, as you can see on tables 11, 12 and 13 that the rankings are very diverse. The tables contain the top 10 riskiest and the bottom 10 safest institutions indicating the sector. However, there are some astonishing points.

V.2.1 Pre-crisis

Start the interpretation with table 11. The SIFIs detected by the $InOut$ measure have quite a lot connections, considering that 113 company were included about 50% of all possible relations deriving from a company and arrive at the company. The ranking enhances the centrality of banks and P/C insurers, but it is surprising that also a health insurer is represented in the ranking, while health insurers are usually treated as a subgroup of life or P/C insurance depending on the classification regime.

Contrasted to the $InOut$ gauge, ΔCoVaR also detected Wells Fargo as a risky firm. The systemically important institutions are banks, only 3 insurers were identified by ΔCoVaR health, a multi-line and a life insurer.

ΔCoVaR identified JP Morgan as a vulnerable institution, which is in accordance with the related measures, MES and $SRISK$.

Considering the marginal expected shortfall outcome, the instability of insurers stands out of the question: property and casualty, life, full line insurers, and the MUV2 reinsurance company are also detected, which shed light on that the sectoral division of insurers can support policymakers.

In the case of SRISK ranking, the dominating role of banks is clear, only the Allianz SE was found, from the insurance sector, posing a high risk.

The "security" rankings are, for me, a more exciting topic, while I am convinced if a company is found safe, it might mean that you might not have the proper toolkit to measure it.

I also should add that the $InOut$ framework detected several companies with zero connections, so it does not deserve to much time, and may it is not proper for this role. I can compare it to $SRISK$, which also includes companies with zero values.

ΔCoVaR described different types of institutions as secure ones, like life, P/C, full line insurers and a reinsurer. Also, banks are detected, Turkiye Garanti Bankasi AS (GARAN) and QN Finansbank AS (GNBFB).

MES also identified small banks and full line insurers as low-risk institutions.

The interconnectedness of individual institutions is expressed on network graph 10. The picture, similar to the ranking scores, suggests a network with several edges and relations.

[12] ΔCoVaR, MES and SRISK were calculated as averages for all institutions.

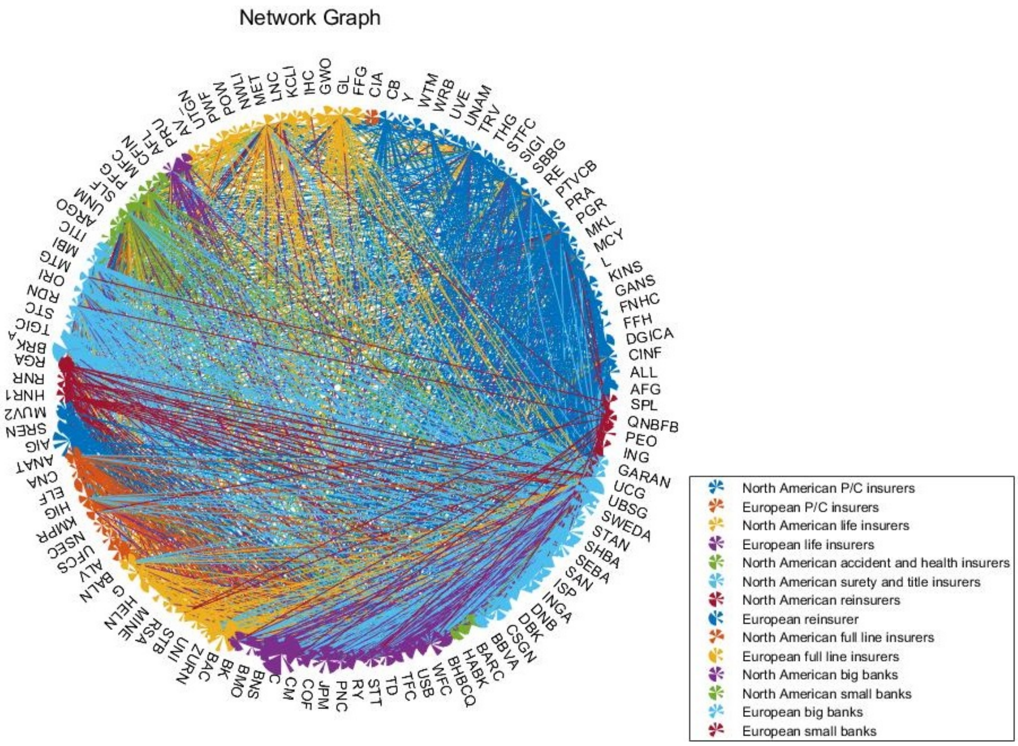


Figure 10: Individual institutions connectedness in the pre-crisis period

V.2.2 Crisis

Considering the period of the financial crisis, it seems to be very surprising, that the banking system caused the financial crisis, but according to the $InOut$ measure, 9 insurers were detected only one bank. Also, an important point is that the SIFIs have halved their exposure during the most severe times of the financial crises. (See also figure 11. Comparing with figure 10, the decrease of connections is easy to see.) This fact may deserve more attention. A possible explanation is that the method proposed by Song and Taamouti (2019) cleared all spurious and indirect edges from the network, and not the direct relations were dominating that period. A further remark is that life insurers were described as the source of the shocks owing to toxic assets.

Not only $InOut$, but also ΔCoVaR identified insurers as a critical part of the financial system and similar to $InOut$, not property and casualty insurers are dominating, but the life insurers.

MES also characterized insurers as SIFIs, but the financial surety and guarantee insurers gave half of the ranking, which traded the toxic assets during the crises. Beverly Hills Bancorp Inc (BHBCQ) was found a vulnerable institution, which is a small American bank. Beverly Hills Bancorp Inc is an example to tiny, but vulnerable firms. An exciting phenomenon is how small institutions can become systemically important. Although it is a rare event, Lin et al. (2018) observed that small, but embedded insurers can become an essential part of financial networks.

$SRISK$ presented a ranking, which most suited my expectations. Thus the banks were the most vulnerable companies between 2007 – 2009.

If you see the bottom of the rankings, similarly to the pre-crisis period, $SRISK$ and $InOut$ rankings are not too useful, while all values are zero. Nevertheless, the zeros signal that some companies remained isolated in the financial sector, which can suggest their safety.

Considering another gauge, ΔCoVaR shows that property and casualty and full line insurers are the most secure institutions. This pattern is also found in the case of MES ranking but completed with banks and one health insurer.

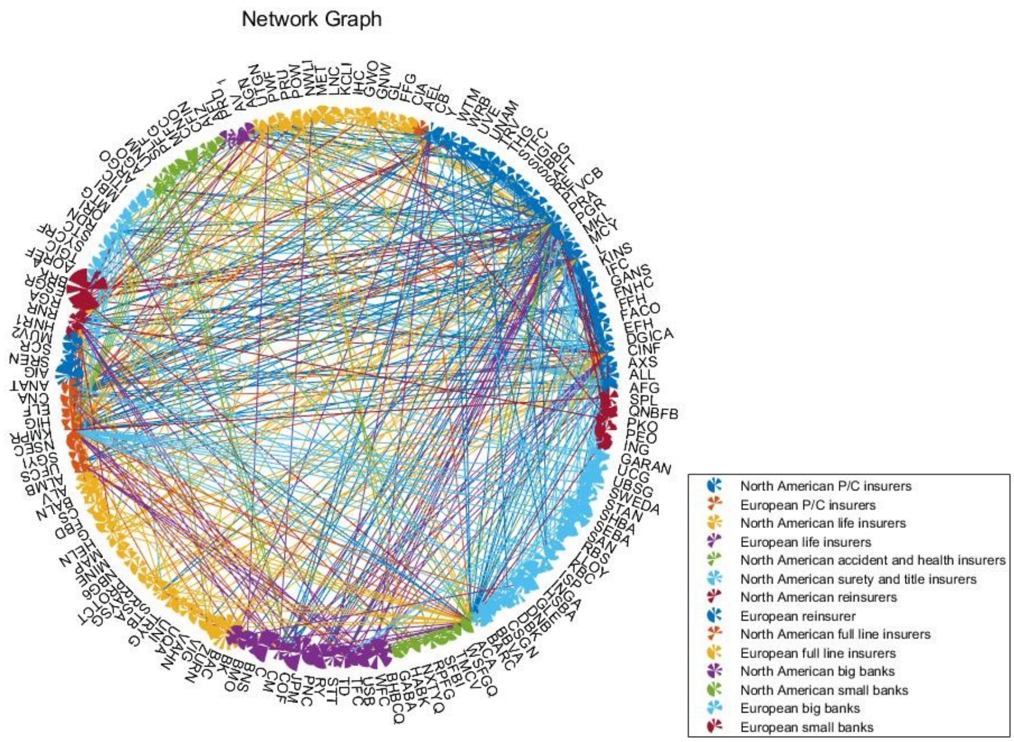


Figure 11: Individual institutions connectedness during the financial crisis

V.2.3 Post-crisis

Table 13 shows that after the financial crises the $Delta$CoVaR, MES and $SRISK$ values dropped. Except the riskiest company identified by MES - WSB Financial Group Inc (WSFGQ) -, WSB Financial Group Inc is operating in the banking industry, and it belongs to the American banks with the lowest market capitalization included in the sample. Similarly, Rainier Pacific Financial Group Inc (RPFG), Nexity Financial Corp (NXTYQ), Georgia Bancshares Inc (GABA) and Beverly Hills Bancorp (BHBCQ) were detected. All in all, five small banks were found to be systemically vulnerable. I think this finding is significant, while authors usually neglect small firms from samples, which can seemingly distort institution rankings.

The $SRISK$ ranking presented that the numbers decreased, but remained higher than in the pre-crisis period, while ΔCoVaR values did not exceed the pre-crisis level.

Only one measure demonstrated a dynamical growth, the $InOut$ framework. The growth expresses the real increase of the connectedness in the financial system, while the relations are calculated by filtering out indirect and spurious edges. Although, the sectoral relations are hard to define, while $InOut$ and ΔCoVaR identified more risky insurers, while in the case of $SRISK$ and MES banks are dominating.

A crucial change is that based on $InOut$, the majority of the safest institutions are not isolated in the network. This is the following sign of the growing connectedness. Nevertheless, the sectoral level is hard to judge without further investigation, but the network graph provides an insight into the possible changes (figure 12), so in the next part, I will focus on the aggregated connectedness.

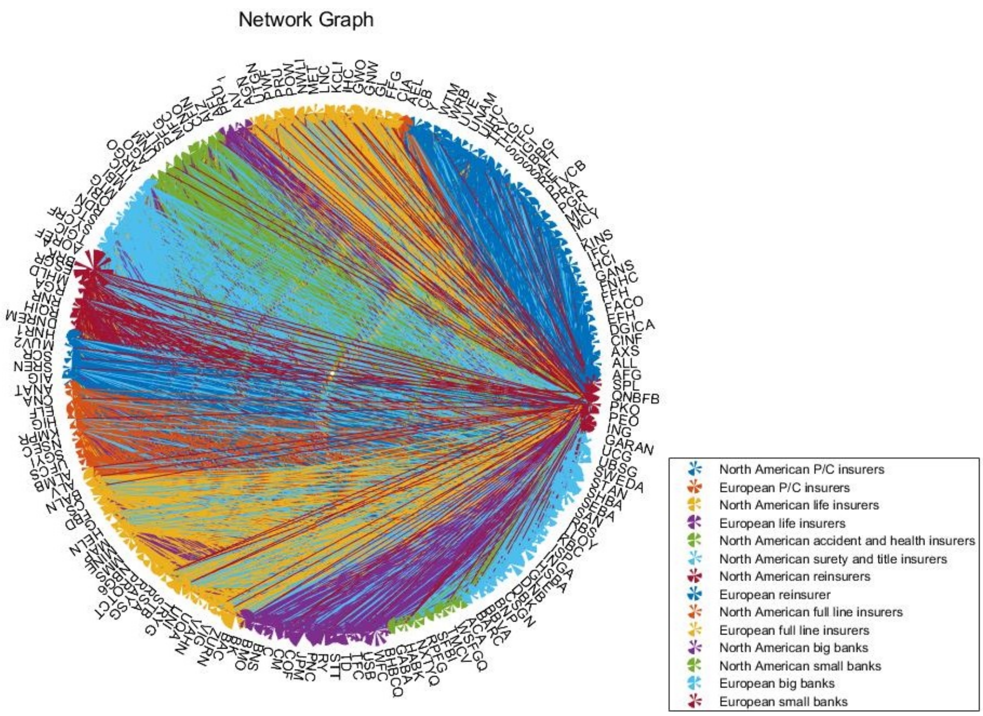


Figure 12: Individual institutions connectedness in the post-crisis period

Top10	**Name**	**InOut**	**Type**	**Name**	**ΔCoVaR**	**Type**	**Name**	**MES**	**Type**	**Name**	**SRISK**	**Type**
1	BMO	103	Bank	WFC	0.00109	Bank	ZURN	0.0224	Full line insurer	UBSG	63 696.1	Bank
2	ALL	101	P/C insurer	USB	0.00075	Bank	UVE	0.0106	P/C insurer	SWEDA	57 449.5	Bank
3	WFC	92	Bank	SAN	0.0007	Bank	RSA	0.0101	Full line insurer	SHBA	57 190.5	Bank
4	STT	91	Bank	PFG	0.00068	Accident and health insurer	MUV2	0.0099	Reinsurer	SEBA	55 690.7	Bank
5	SLF	88	Accident and health insurer	JPM	0.00066	Bank	JPM	0.0096	Bank	JPM	45 244.4	Bank
6	RY	88	Bank	HIG	0.00065	Full line insurer	INGA	0.0089	Bank	INGA	35 199.6	Bank
7	PGR	85	P/C insurer	GL	0.00065	Life insurer	GANS	0.0085	P/C insurer	DBK	34 730	Bank
8	L	85	P/C insurer	COF	0.00061	Bank	COF	0.0085	Bank	CSGN	31 228.2	Bank
9	CM	84	Bank	C	0.00056	Bank	AV_	0.0084	Life insurer	BARC	27 225.4	Bank
10	CINF	82	P/C insurer	BAC	0.00056	Bank	ALV	0.008	Life insurer	ALV	25 912.4	Full line insurer
Down10	**Name**	**InOut**	**Type**	**Name**	**ΔCoVaR**	**Type**	**Name**	**MES**	**Type**	**Name**	**SRISK**	**Type**
1	UNAM	1	P/C insurer	CIA	-0.00013	Life insurer	UNI	-0.0281	Full line insurer	WFC	0	Bank
2	UFCS	1	Full line insurer	GARAN	-0.0002	Bank	SBBG	-0.0071	P/C insurer	UFCS	0	Full line insurer
3	PEO	1	Bank	GWO	-0.00002	Life insurer	QNBFB	-0.0031	Bank	TGIC	0	Surety and title insurer
4	NWLI	0	Life insurer	HELN	-0.00004	Full line insurer	NSEC	-0.0025	Full line insurer	SPL	0	Bank
5	NSEC	0	Full line insurer	HNR1	-0.00004	Reinsurer	MINE	-0.0007	Full line insurer	RNR	0	Reinsurer
6	MINE	0	Full line insurer	IHC	-0.00006	Life insurer	HELN	-0.0005	Full line insurer	RGA	0	Reinsurer
7	ITIC	0	Surety and title insurer	QNBFB	-0.00006	Bank	HABK	-0.0003	Bank	PEO	0	Bank
8	ELF	0	Full line insurer	RSA	-0.00006	Full line insurer	GWO	-0.0001	Life insurer	KMPR	0	Full line insurer
9	CFIN	0	Accident and health insurer	UNAM	-0.00008	P/C insurer	ELF	-0.0001	Full line insurer	BRK_A	0	Reinsurer
10	ARGO	0	Surety and title insurer	UVE	-0.0001	P/C insurer	BHBCQ	-0.0001	Bank	ANAT	0	Full line insurer

Table 11: SIFIs and safest institutions based on different measures in the pre-crisis period

Top10	Name	InOut	Type	Name	ΔCoVaR	Type	Name	MES	Type	Name	SRISK	Type
1	UTGN	57	Life insurer	WFC	0.00198	Bank	TGIC	0.0599	Surety and title insurer	SEBA	147 677.1	Bank
2	RNR	52	Reinsurer	UNM	0.00188	Accident and health insurer	SYCRF	0.0514	Surety and title insurer	RBS	140 593.3	Bank
3	PRA	51	P/C insurer	UCG	0.00179	Bank	RDN	0.0444	Surety and title insurer	INGA	135 685	Bank
4	PGR	48	P/C insurer	PWF	0.00168	Life insurer	MTG	0.0416	Surety and title insurer	HSBA	132 208.2	Bank
5	L	42	P/C insurer	PRU	0.0016	Life insurer	MBI	0.0359	Surety and title insurer	DBK	122 035.9	Bank
6	KMPR	40	Full line insurer	POW	0.00154	Life insurer	LNC	0.0356	Life insurer	C	113 717.3	Bank
7	CNA	37	Full line insurer	PGR	0.00153	P/C insurer	HIG	0.0349	Full line insurer	BARC	102 950.6	Bank
8	CINF	36	P/C insurer	JPM	0.00153	Bank	GNW	0.033	Life insurer	ACA	101 557.5	Bank
9	AFG	33	P/C insurer	CB	0.00152	P/C insurer	BHBCQ	0.0314	Bank	UBSG	99 005.4	Bank
10	ACA	24	Bank	ACA	0.0015	Bank	AOREF	0.0313	Reinsurer	SWEDA	94 914.1	Bank
Down10	**Name**	**InOut**	**Type**	**Name**	**ΔCoVaR**	**Type**	**Name**	**MES**	**Type**	**Name**	**SRISK**	**Type**
1	VAHN	0	Full line insurer	PROTCT	-0.0004	P/C insurer	SGYI	0.0026	Full line insurer	Y	0	P/C insurer
2	SGYI	0	Full line insurer	NSEC	-0.00026	P/C insurer	SFBI	0.0021	Bank	TRYG	0	Full line insurer
3	SFBI	0	Bank	GANS	-0.00024	P/C insurer	SBBG	0.0017	P/C insurer	THG	0	P/C insurer
4	RBS	0	Bank	FNHC	-0.00018	P/C insurer	RAYSG	0.0007	Full line insurer	STFC	0	P/C insurer
5	PROTCT	0	Full line insurer	FACO	-0.00016	Full line insurer	PROTCT	-0.0309	Bank	SAFT	0	P/C insurer
6	NBG6	0	Full line insurer	EFH	-0.00015	P/C insurer	NBG6	-0.0045	Full line insurer	RNR	0	Reinsurer
7	C	0	Bank	VAHN	-0.00009	Full line insurer	HABK	-0.002	Bank	RAYSG	0	Full line insurer
8	BNS	0	Bank	UNAM	-0.00007	Full line insurer	ELF	-0.0011	Full line insurer	ITIC	0	Surety and title insurer
9	BARC	0	Bank	STB	-0.00007	Full line insurer	EFH	-0.0006	P/C insurer	CIA	0	Life insurer
10	ALV	0	Full line insurer	RAYSG	-0.00003	Full line insurer	CFIN	-0.0003	Accident and health insurer	BRK_A	0	Reinsurer

Table 12: SIFIs and safest institutions based on different measures during the financial crisis

Top10	**Name**	**InOut**	**Type**	**Name**	**ΔCoVaR**	**Type**	**Name**	**MES**	**Type**	**Name**	**SRISK**	**Type**
1	TRYG	173	Full line insurer	UNM	0.00073	Accident and health insurer	WSFGQ	0.069	Bank	BNP	123 951.8	Bank
2	SLF	171	Accident and health insurer	PRU	0.0006	Life insurer	TGIC	0.0258	Surety and title insurer	BARC	118 093	Bank
3	SHBA	169	Bank	PNC	0.00056	Bank	SGYI	0.0242	Full line insurer	BAC	111 133.4	Bank
4	RSA	168	Full line insurer	PFG	0.00053	Accident and health insurer	SFBI	0.0203	Bank	ACA	107 900.2	Bank
5	PWF	164	Life insurer	MET	0.00051	Life insurer	RPFG	0.014	Bank	SHBA	91 477.4	Bank
6	POW	162	Life insurer	LNC	0.0005	Life insurer	RDN	0.0134	Surety and title insurer	SEBA	83 698.9	Bank
7	MET	158	Life insurer	JPM	0.0005	Bank	NXTYQ	0.0105	Bank	HSBA	80 650.3	Bank
8	HNR1	158	Reinsurer	BRK_A	0.00047	Reinsurer	KINS	0.01	Reinsurer	GLE	72 214.2	Bank
9	DBK	158	Bank	BK	0.00047	Bank	GABA	0.0099	Bank	DNB	71 469.8	Bank
10	BK	157	Bank	BAC	0.00046	Bank	BHBCQ	0.0094	Bank	DBK	69 589.9	Bank
Down10	**Name**	**InOut**	**Type**	**Name**	**ΔCoVaR**	**Type**	**Name**	**MES**	**Type**	**Name**	**SRISK**	**Type**
1	UTGN	2	Life insurer	VIG	-0.00008	Reinsurer	UIHC	-0.0236	P/C insurer	Y	0	P/C insurer
2	UNAM	1	P/C insurer	UNAM	-0.00006	P/C insurer	TMCV	-0.005	Bank	TRYG	0	Full line insurer
3	SGYI	1	Reinsurer	UIHC	-0.00006	P/C insurer	SBBG	-0.004	Reinsurer	SAFT	0	P/C insurer
4	NXTYQ	1	Bank	QNBFB	-0.00003	Bank	NSEC	-0.002	Full line insurer	RNR	0	Reinsurer
5	NSEC	1	Full line insurer	NSEC	-0.00003	Full line insurer	NBG6	-0.0012	Full line insurer	RE	0	P/C insurer
6	NBG6	1	Full line insurer	KINS	-0.00002	P/C insurer	HABK	-0.001	Bank	PROTCT	0	Full line insurer
7	MMS	1	Reinsurer	FACO	-0.00002	P/C insurer	GANS	-0.0008	Reinsurer	PRA	0	P/C insurer
8	GABA	1	Bank	ELF	-0.00002	Full line insurer	FACO	-0.0003	Reinsurer	ITIC	0	Surety and title insurer
9	BHBCQ	0	Bank	ALMB	-0.00001	Full line insurer	ELF	-0.0002	Full line insurer	ESGR	0	Reinsurer
10	AOREF	0	Reinsurer	TGIC	-0.000005	Life insurer	AOREF	-0.0002	Reinsurer	BRK_A	0	Reinsurer

Table 13: SIFIs and safest institutions based on different measures during after the crisis

V.3 Connections among sectors

The analysis of the sectors is not only useful to understand the vulnerability of the financial system but also can provide information for policymakers. At the same time, this type of differentiation can be a proper clustering approach of the financial institutions, which results separated groups based on business activities. Considering sectoral level relations also suggest the direction of systemic level connectedness. For example, compare network graphs calculated under and before the crisis, respectively, under and after the great recession in 2008.

V.3.1 Pre-crisis

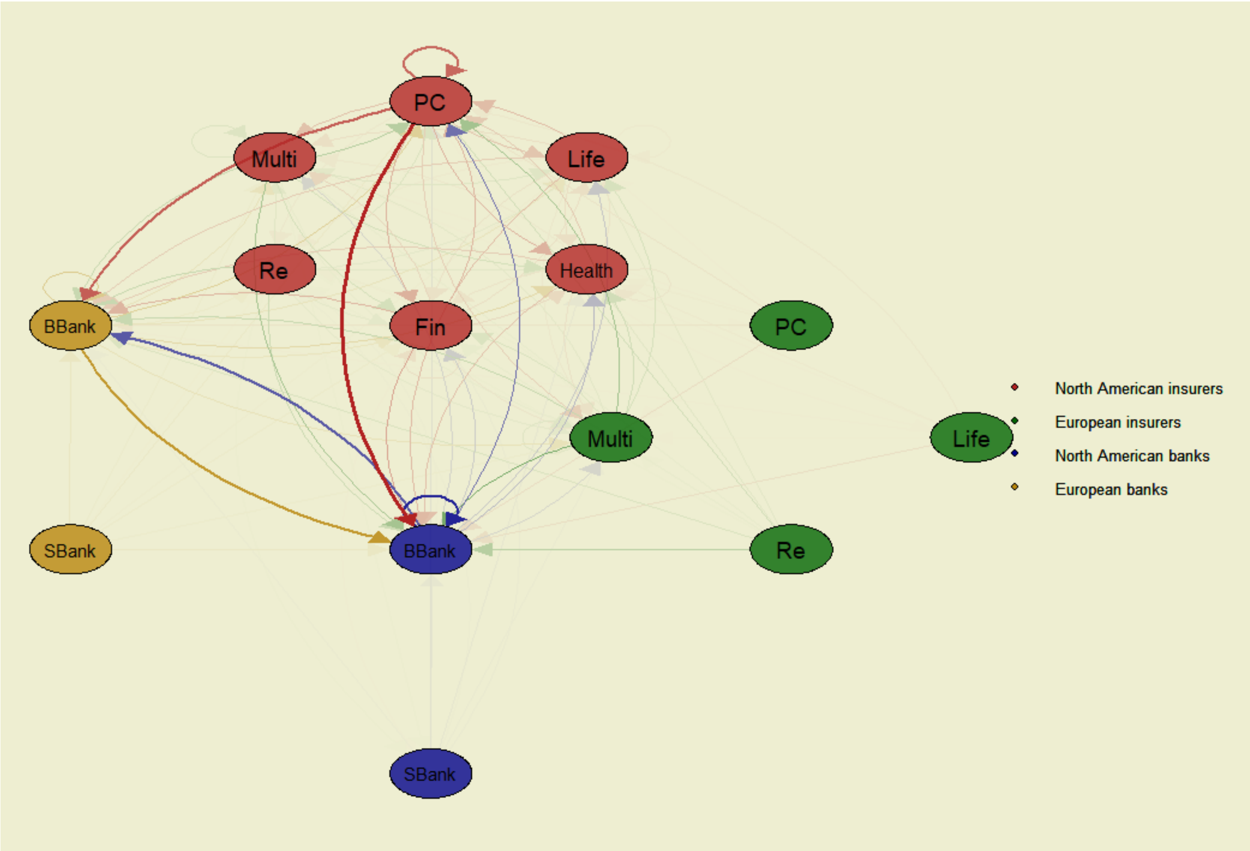
I plotted the European and North American banking and insurance sector connectedness in the first decade of the 21^{th} century on figure 13. On network graph, four clusters are isolated: European insurers (green), North American insurance sector (red), European banks with the low and high market capitalization (gold), and North American big and small banks (blue).

The network shows precisely that the North American insurance sector and the highly capitalized banks were the most related to each other, while smaller banks seem to be isolated. The European insurance industry is also hardly related to the other branches, except the European multi-line insurers. The European full line insurers presented the 60% of the European insurers included in the pre-crisis sample, which explains their importance compared to the other sectors. The European reinsurance and P/C insurance sector had a weak connection with other sectors, like the European reinsurers reduced the exposure of North American Life insurers and banks.

The chart presents that, on the American and Canadian market, more separate insurance sectors can be distinguished, then on the European market. The property and casualty insurance sector related mostly to the banking industry. The surety and financial guarantee insurers have several linkages with inter-sectoral and external partners. However, this sector mainly absorbs shocks instead cause them similar to the health and accident insurers. Compared to other branches, North American life insurers are the less interconnected due to my calculation in the pre-crisis time horizon. Lastly, the leading role of the banking sector on both continents is convincing, and the North American and European financially strong banks are deeply interconnected.

V.3.2 Crisis

During the time of the financial crisis, expectedly would increase the number of connections. Nevertheless, figure 14 shows a contrary phenomenon. The number of linkages dropped greatly compared to the previous period. As far as I know, no similar experience was former reported. The dramatic decrease of linkages can only be explained if the network



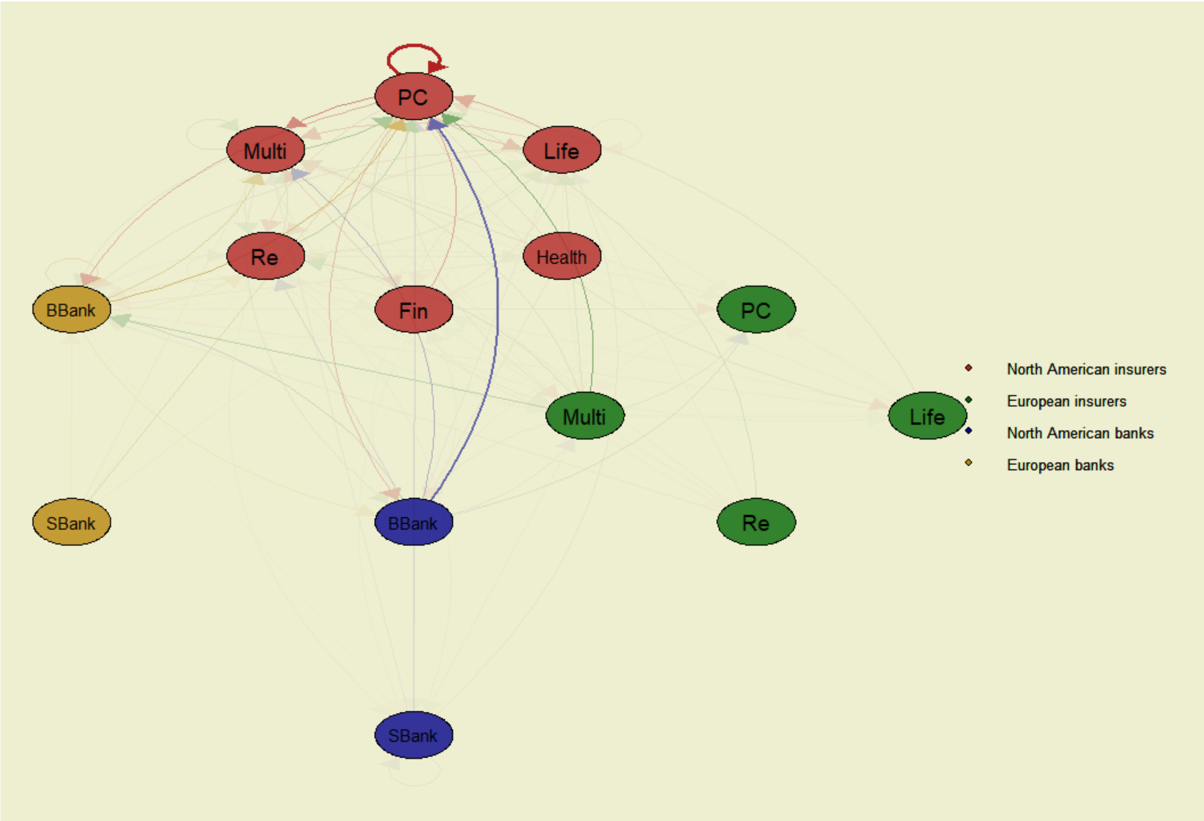
Notes: *PC* means property and casualty insurers, *Multi* symbolizes multi-line or full line insurers, *Re* abbreviates reinsurers, *Fin* summarizes surety and financial guarantee insurers, *Life* naturally life insurers, *SBank* categorizes small banks and lastly *BBank* banks with higher market capitalization.

Figure 13: Network graph in the pre-crises period indicating, financial sectors

was full of indirect and spurious edges, which was cleared by the testing process. I also remark that a little distortion can be derived from the PCA method, while it serves as a rule of thumb, that the number of observations should exceed the quintuple of the number of institutions. As 154, companies are included, and only 152 were used to the principal component analysis, so about 760 observations needed, while I had only 522 during the static analysis. However, considering the results of the Monte Carlo simulation (Appendix C.2) the solution is robust enough for economic analysis.

After the methodological detour, returning back to figure 14, I can state that European life insurers, reinsurers and P/C insurer connectivity have become weaker. Similar to European and North American small banks. Nevertheless, seemingly the connections between the European and North American market is reduced. The linkages diminished mainly between large banks, which contradicts Diebold and Yilmaz (2015) observation, who pointed out that during the crisis, the American banks Granger-caused European institutions. Only the North American property and casualty insurers have a more substantial impact on the highly capitalized European banks.

North American P/C industry seems to be the central sector during the great depression



Notes: *PC* means property and casualty insurers, *Multi* symbolizes multi-line or full line insurers, *Re* abbreviates reinsurers, *Fin* summarizes surety and financial guarantee insurers, *Life* naturally life insurers, *SBank* categorizes small banks and lastly *BBank* banks with higher market capitalization.

Figure 14: Network graph during the financial crisis, indicating financial sectors

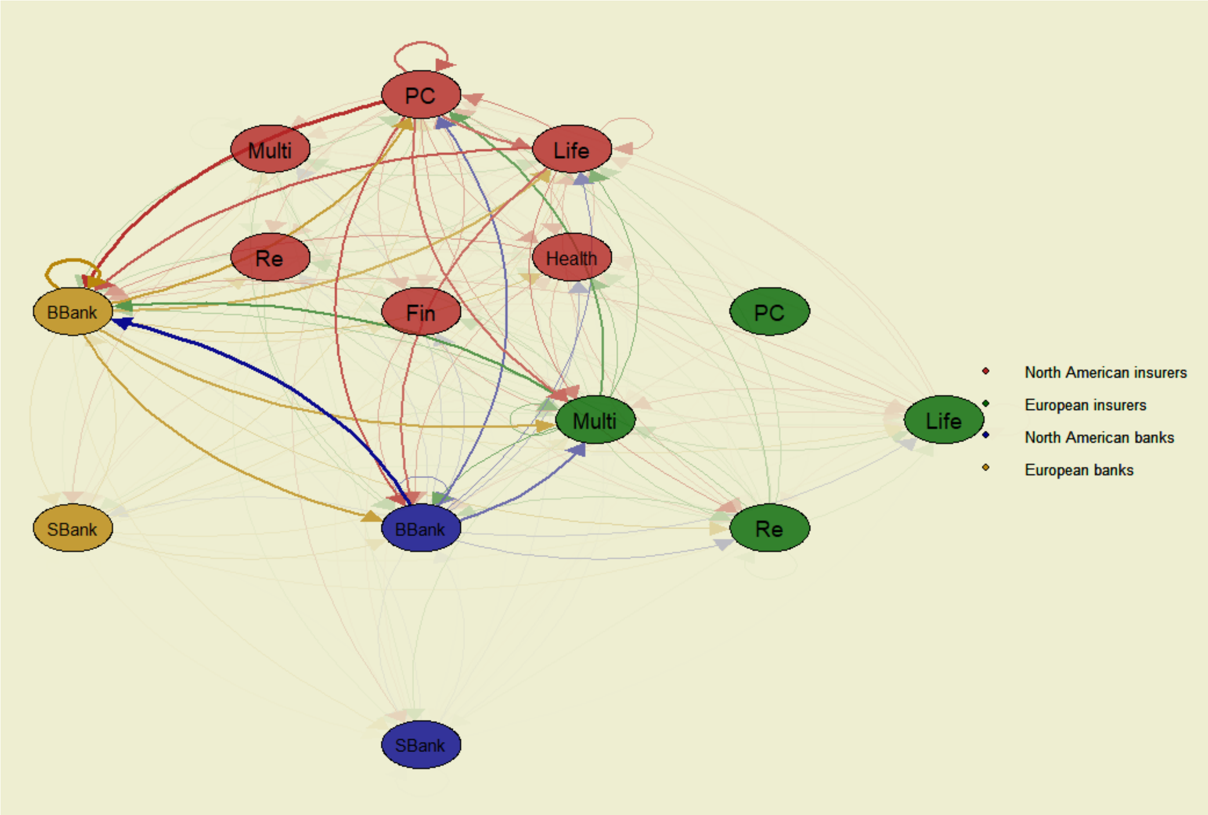
in 2008, which can be explained by the contra-cyclical nature of the insurance sector and the more developed unemployment insurance market and growing expenses.

V.3.3 Post-crisis

After the distressed period, the connectivity of the insurance and banking network started to grow fast. The highest acceleration could be found in the European life, multi-line, reinsurance sector and the smaller Eastern-European banks built up several new relations.

In the North American insurance sector, the volume of growth was smaller- except in life insurance-, but the basis was higher before the turmoil period. The North American life insurers quadrupled their connectivity, which was the most outstanding evaluation.

Of course, financially strong banks carried out also an expansion on both continents, which contributed to the growing interconnectedness. European banks deepened its linkages with European full-line insurers and North American life insurers. While North American banks Granger-causes European multi-line insurers and North American P/C insurers.



Notes: *PC* means property and casualty insurers, *Multi* symbolizes multi-line or full line insurers, *Re* abbreviates reinsurers, *Fin* summarizes surety and financial guarantee insurers, *Life* naturally life insurers, *SBank* categorizes small banks and lastly *BBank* banks with higher market capitalization.

Figure 15: Network graph in the post-crises period indicating, financial sectors

V.4 Industrial connectedness

The changes in the financial sectors are summarized in table 15. The table confirms the massive drop in the interconnectedness of the insurance and banking industry, while a fast rebuild of the network can be observed after the turmoil. During the analysed about 20 years period, the European insurance sector produced the most substantial growth, followed by the North American banking and insurance industry. The smallest expansion was seen in the European banking business, maybe thanks to the sovereign debt crisis.

V.5 System connectedness

The institutional and sectoral level investigation suggested that a huge drop was observed in the number of connections during the financial crisis. For validation, I calculated the number of relations using the DCI measure in all periods, and I contrast it to the number of real edges detected by the approach proposed by Song and Taamouti (2019).

Table 16 summarizes the identified causalities. I indicated the possible linkages, while the number of analysed institutions is changing time after time. In the pre-crisis period, I detected 2023 linkages, while the linear-Granger causality test more than 12000, so the ratio

	Pre-crises	Crisis	Post-crisis
North American P/C insurers	0.149	0.039	0.205
European P/C insurers	0.138	0.056	0.206
North American life insurers	0.086	0.015	0.325
European life insurers	0.047	0.016	0.401
North American accident and health insurers	0.202	0.008	0.332
North American surety and title insurers	0.167	0.018	0.213
North American reinsurers	0.143	0.039	0.199
European reinsurer	0.112	0.009	0.403
North American full line insurers	0.138	0.045	0.15
European full line insurers	0.13	0.011	0.243
North American big banks	0.288	0.019	0.38
North American small banks	0.058	0.009	0.061
European big banks	0.205	0.02	0.361
European small banks	0.038	0.01	0.178

Table 14: Financial sectors connectedness measured by $InOut_k$

	Pre-crises	Crisis	Post-crisis
European insurance sector	0.116	0.013	0.284
North American insurance sector	0.142	0.029	0.234
European banking industry	0.261	0.016	0.269
North American banking industry	0.161	0.017	0.321

Table 15: Connectedness of financial industries measured by $InOut_k$

of the indirect or spurious causalities is about 84%, which is an enormous proportion and indicates that several non-real relationships exist in the banking and insurance sectors.

Furthermore, there are less true, but more false linkages during the financial crises, while the interconnectedness surprisingly decreased. The proportion of indirect and spurious edges is approximately 97%, which is higher with 13%-point compared to the previous period. Probably, the non-real relations played a crucial role in the amplification of the risk spillovers.

Nevertheless, in the post-crisis term, the trend has changed. The number of real connections was multiplied by 13 times, while the indirect and spurious connections increased moderately. This phenomenon caused that the proportion of non-real linkages dropped

26%-point and became less than in the pre-crisis period, which suggests a more organic growth. However, the division of relationships is still unbalanced.

	Pre-crises	Crisis	Post-crisis
Real connections	2 023	522	6 771
All detected connections	12 483	20 372	23 718
Possible connections	12 656	23 562	25 760
Proportion of non-real connections	0.838	0.974	0.715

Table 16: System connectedness and real edges in different periods

Also, a significant difference can be observed considering the DCI based on Billio et al. (2012) and its extension DCI*, deriving from Song and Taamouti (2019). As the figure **??** shows, the measure of Billio et al. (2012) detected a dense network in the pre-crisis period with 98.6% connectedness, while based on true linkages the DCI* shows only 16%. A typical problem arises that without historical experience, it is hard to judge that the outcomes signal dense or rarely connected network, despite that the DCI measure is normed between 0 and 1.

During the financial crisis (2007 – 2009), the level of connectedness decreased according to both approaches, but DCI demonstrates ca. 87%, while DCI* only about 2%. However, the more exciting thing is that in the post-crisis period, the DCI* demonstrated a more dynamic growth compared to DCI and higher value, then before 2007. At the same time, DCI remained less than in the pre-crisis period.

The results suggest that the number of indirect and spurious causalities increases during distressed periods, which may amplify risk spillovers in the financial sector.

V.6 Robustness check

The proper systemic risk gauges should be able to signal the future downturns, also work as early warning indicators. To test this feature of the $InOut_k$ measure, I follow the approach of Hué et al. (2019) and Sedunov (2016) and build regression models to check the loss prediction performance of my measure. The purpose is to predict the ranking of downside log-returns of the institutions in different periods and make a forecast on the forthcoming term to validate the results. I compassed the robustness check in two periods. Firstly, I built models on the pre-crisis data and validated on crisis data. Furthermore, I also fitted regressions on the crisis data and checked its performance on the loss of the forthcoming 250 days, while it would not be proper to validate the outcome on a much larger post-crisis period.

There is one point of the robustness check, which differs from the literature and the framework of Hué et al. (2019). Also, I do not use linear regression for the validation, but

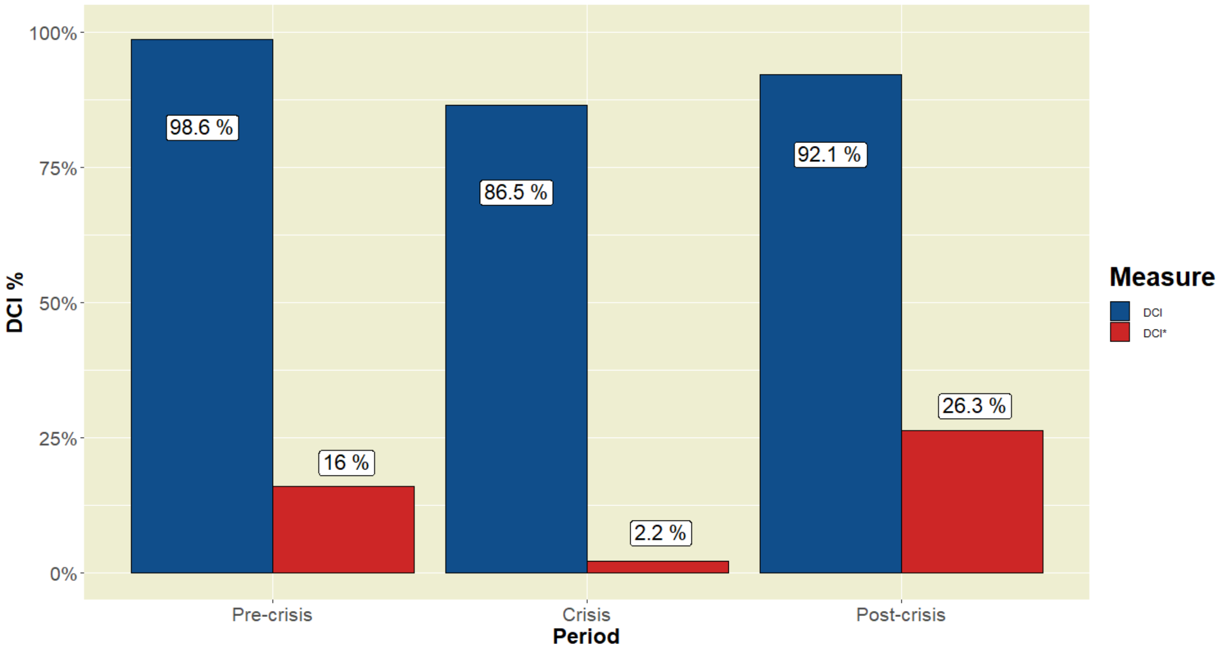


Figure 16: Connectedness of the financial system in different periods

ordinal logistic regression (McCullagh 1980). Namely, I will predict the ranking of the highest losses, not the amount of loss in accordance with Hué et al. (2019), but in the rankings, the order of the numbers contains extra information. Moreover, the linear regression modell predicts continuous variables, not whole numbers, which is statistically not correct. So, I chose the ordinal logistic regression approach, and instead of R squared I used AUC[13] (Area Under the Curve) to compare different modells.

The testing procedure is organised as follows (Hué et al. 2019, p.103-104):

1. The losses are calculated as the average return under a given threshold ($\delta = -3\%$) for all institutions[14].

$$\mathrm{Perf}_k = \frac{1}{n}\sum_{t=1}^{T_c} y_t^k Z_t^k, \tag{57}$$

[13]Thus, the ordinal logistic regression predicts ordered classes, R squared is not meaningful in this case. Despite the AUC quantifies the fitness of the modell calculating the area under the ROC curve. ROC curve illustrates the proportion of correctly and erroneously categorised elements of the data set considering different cut values. In poor modells, there is no distinction between different groups, and the ROC curve becomes the identity mapping on the unit square, so AUC deviates from 0.5 to 1. The higher the AUC is, the better the forecast is.

[14]The construction works similar to the expected shortfall.

where T^c is the number of days, y_t^k the daily log return of institution k, while

$$Z_t^k = \begin{cases} 1, \text{if } y_t^k \leq \delta, \\ 0 \text{ otherwise.} \end{cases} \tag{58}$$

2. The institutions are ranked based on the average losses. Moreover, the proportion of incoming edges (In_k), the caused causalities (Out_k), and the combination of them ($InOut_k$) is also transformed into rankings. The rank numbers generated from the gauges are signaled with square brackets.

   I also involved in the regression the measures calculated based on my correction and the original results based on the approach of Billio et al. (2012) using FWER-correction.[15]

3. I run different ordinal logistic regression modells, for the illustration I depict the general form of the ordinal linear regression:

$$\begin{aligned} \ln(odds_{>0}) &= z_0 + \sum_i^V b_i \Xi_i \\ &\vdots \\ \ln(odds_{>U-2}) &= z_{U-2} + \sum_i^V b_i \Xi_i \end{aligned} \tag{59}$$

   where $odds_{>j} = \frac{\mathbb{P}(Y>j)}{\mathbb{P}(Y\leq j)}$, which means the proportion of the cumulative odds of the categories is greater than j and the cumulative odds of the groups less or equal than j ($j = 0, ..., U-2$). Totally, there are U categorizes, and the probability of the groups is estimated. $\Xi_i, i = 1, ..., V$ are the explaining variables. I used $[In_k]$, $[In_k]$, $[Out_k]$, $[InOut_k]$, $[In_k^*]$, $[Out_k^*]$ and $[InOut_k^*]$ as explaining variable, which denotes the rank number generated from the mentioned measures.

4. The modells are validated on the out-of-sample by the calculation of AUC.

Table 17 summarizes the regression outputs for the pre-crisis period validated on the crisis losses. The aim of the robustness check is not to find the best modell predicting future negative returns, but the confirmation of the explaining power of the calculated measures. I computed 7 regressions, in the first 6, I tested the significance of the individual gauges, which resulted that all of my measures were significant, while only Out_k from Billio et al. (2012). I also calculated an extended modell, including In_k, Out_k, In_k^* and Out_k^* variables. I did not involve$InOut_k$ and $InOut_k^*$., while $InOut_k$ measures are calculated as the sum of In_k and Out_k after some correction, and it would cause multicollinearity in the regression. I present

[15]A little star in the top-right corner indicates my measures.

Measure \ Modell	1	2	3	4	5	6	7
In_k^*	0.0326*** (4.57)	-	-	-	-	-	0.0246*** (2.85)
Out_k^*	-	0.0223*** (4.136)	-	-	-	-	0.0119** (1.868)
$InOut_k^*$	-	-	0.0267*** (4.762)	-	-	-	-
In_k	-	-	-	-0.0087 (-1.267)	-	-	0.0042 (0.567)
Out_k	-	-	-	-	0.026*** (2.77)	-	0.0175** (1.813)
$InOut_k$	-	-	-	-	-	-0.0027 (-0.397)	-
AUC	0.5	0.867	0.9	0.801	0.812	0.79	0.824

Notes: In_k^*, Out_k^* and $InOut_k^*$ were calculated based on my modifications. The table contains the coefficient of the different measures in the ordinal logistic regressions. In parenthesis, I indicated the value of the t-statistics, while the little signs express the magnitude of the p-values. *** $p < 0.01$ ** $0.01 \leq p < 0.05$ * $0.05 \leq p < 0.1$

Table 17: Ordinal logistic regression output calculated on the pre-crises data and validated on the crises losses, $\delta = -3\%$

this modell to contradict Hué et al. (2019), who did not find these measures significant in the extended regression.

The main findings can be seen in the last row of the table 17. The highest AUC value belongs to the modell containing only the $InOut_k^*$ variable, which supports the statement that the $InOut_k$ measure is proper for the prediction of high losses. The result also confirms the expectations of Hué et al. (2019), that filtering out non-real relations leads to an adequate measure for systemic risk analysis.

The second part of the test focused on the crisis data and tried to forecast the losses that occurred in the next 250 days. The procedure is the same as the former. The table 18 describes the coefficients, test-statistics and significance values. A clear change is that only three significant individual modell exists (at the common 5% significance level). Both $InOut_k^*$ and Out_k are quite efficient modells, but the Out_k^* performed the best. However, in the joint modell only the Out_k variables are significant. Considering the appropriate individual modells, modell 3 has a better performance in prediction in comparison to modell 5. The difference is not too high, but the results confirm that in other periods of the business cycle, the predicting power remains at least so punctual as in the case of Out_k.

All in all, the robustness tests verified the predicting power of the framework independently from the business cycle.moreover, in the forecast of the crisis losses, my measure was found more appropriate.

Measure \ Modell	1	2	3	4	5	6	7
In_k^*	0.0085 (0.887)	-	-	-	-	-	0.00298 0.286
Out_k^*	-	0.0095*** (2.756)	-	-	-	-	0.00832** 2.225
$InOut_k^*$	-	-	0.0078** (2.273)	-	-	-	-
In_k	-	-	-	-0.0014 (-0.438)	-	-	0.00106 0.314
Out_k	-	-	-	-	0.0063** (2.257)	-	0.00581** 1.913
$InOut_k$	-	-	-	-	-	0.0024 (0.751)	-
AUC	0.742	0.912	0.894	0.769	0.884	0.859	0.893

Notes: In_k^*, Out_k^* and $InOut_k^*$ were calculated based on my modifications. The table contains the coefficient of the different measures in the ordinal logistic regressions. In parenthesis, I indicated the value of the t-statistics, while the little signs express the magnitude of the p-values. *** $p < 0.01$ ** $0.01 \leq p < 0.05$ * $0.05 \leq p < 0.1$

Table 18: Ordinal logistic regression output calculated on the crises data and validated on the next 250 days losses, $\delta = -3\%$

VI Further research

My investigation focused on a large financial network utilizing causality-analysis in institutional, sectoral and industrial level to provide a broader overview of the relations of insurers and banks. Despite the detailed research, my work offers three primary ways for further extensions.

The first possible research way is based on data complexity. I included an extended database in my investigation, but I reflected only on the European and North American markets. Nevertheless, the most important companies and developed regions were covered, involving other institutions that can result interesting observations based on local regulation or particular market structure (e.g. China), as presented by Lin et al. (2018).

Moreover, other types of institutions can also be examined following the stepwise aggregation approach, which can help recover the exposure of one sector to the other one. So, not only the financial sector can be relevant for network analysis, but others like technological companies or the FMCG industry with long supply chains.

As I have pointed out in section III.6., I did not put a high emphasis on dealing with noisy data, which can cause distortions in the calculations highlighted by Nucera et al. (2016), who proposed a potential method to solve this problem. It suggests the combination of measures to filter out noise, which proposes another path for academic work considering noise filtering, model combination, or hybrid modelling to improve the applied framework.

Secondly, further methodological aspects can be included in my analysis. The financial econometrics literature usually operates with linear models, nevertheless, financial data are usually non-linear. So, non-linear Granger-causality analysis is a relevant extension for my modelling results in order to reveal further effects, which are failed to detect by linear models. Accordingly, a robustness check could confirm the sensitivity of the approach.

As the last point, I must mention the topic standing on the crossroads of data linked questions and methodological considerations, namely, an exciting work could be comparing the results deriving from tail-return and CDS price analysis. While CDS prices cover the most information from the state of the network (Rodríguez-Moreno and Peña 2013), until returns are characterized as a less informative source of systemic risk, but the tail behaviour of returns might present some similar or different characteristics compared to CDS prices.

VII Summary

My research focused on the European and North American insurance and banking industries to reveal true connections among individual firms, subsectors and branches. My investigation covered three fields: literature review, methodological framework and empirical analysis.

In the first part, I summarized the empirical papers according to the level of aggregation (individual institutions, sectors, industries). I pointed out that this is a different categorization of statistical methodologies and practical analysis. The literature review revealed the unanswered questions related to the insurance sector. Also, the insurance sector was usually treated as a homogenous branch of the financial sector, which stands in contrast to the business lines of insurers. Also, the insurers operate in more or less disjunct field. (Some overlapping activities, of course, exist,e.g. life insurance vs. health insurance. However, life and P/C insurance differs very much.)

So, I decided to understand the connections between insurance sectors and their relation with the banking industry. Moreover, I tried also to characterize SIFIs in both branches.

For the investigation, I tried to reconstruct the real network of financial firms based on pair-wise Granger causality testing. Although the original test does not handle the indirect and spurious effect existing between companies, so firstly, I tried to utilize the leave-one-out approach of Hué et al. (2019). Which idea was perfect, but the practical application via ΔLGC measure was found methodologically and economically inadequate (section IV.1.4). So, I extended the testing procedure with the methodology of Song and Taamouti (2019), also I used principal component analysis to represent the not involved institutions in the pairwise-causality testing removing indirect and spurious effects from the network.

The improved Granger-causality test was applied to insurance companies and banks between 2001 and 2019, dividing the timeline into three periods: pre-crisis (2001 – 2007), crisis (2007 – 2009) and post-crisis (2009 – 2019). This division served to compare the results in different periods of the business cycle.

In the pre-crisis period, large banks and North American insurers were detected as important edges of the financial network, while the European market was less related thanks to the lot multi-line insurers, which were included in the sample and which operates in more businesses, so the shock was held inside the sector.

The time of the financial crisis presented the most surprising results. The connectedness of the financial network dropped dramatically, which stands in contrast with the common sense in the literature. However, the phenomenon means that during turmoils, the indirect and spurious cause problems in the solvency of companies.

A further interesting result was found, that small a bank was detected as SIFI in accordance with Lin et al. (2018), while the firms with low market capitalization are in general excluded from the researches, which can create distortions during distressed periods.

Moreover, the number of connections between European and North American banks became weaker in spite of the finding of Diebold and Yilmaz (2015). This is also in accordance with the effect of non-real causalities.

Finally, the post-crisis analysis demonstrated a clear up-warding trend in connectedness statistics. The connectedness of banking and insurance industries became equalized, but the leading role of North American banks stand out of the question. Accordingly, in the insurance sector, life and accident and health insurers showed the most dynamical growth in connectedness.

I believe that this paper can serve as stop-gap research in the insurance sector, shedding light on the importance of the detection of SIFIs, inter-sectoral linkages, and connections among industries in order to provide a full overview about the movements and risk of the financial sector.

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Appendix

A Data

Ticker	Company name	Location	Sector	Pre-crisis	Crisis	Post-crisis
AFG	American Financial Group Inc	North America	P/C insurer	x	x	x
ALL	Allstate Corp	North America	P/C insurer	x	x	x
AXS	Axis Capital Holdings Ltd	North America	P/C insurer		x	x
CINF	Cincinnati Financial Corp	North America	P/C insurer	x	x	x
DGICA	Donegal Group Inc	North America	P/C insurer	x	x	x
EFH	Echelon Financial Holdings Inc	North America	P/C insurer		x	x
FACO	First Acceptance Corp	North America	P/C insurer		x	x
FFH	Fairfax Financial Holdings Ltd	North America	P/C insurer	x	x	x
FNHC	FedNat Holding Co	North America	P/C insurer	x	x	x
GANS	GAINSCO Inc	North America	P/C insurer	x	x	x
HCI	HCI Group Inc	North America	P/C insurer			x
IFC	Intact Financial Corp	North America	P/C insurer		x	x
KINS	Kingstone Cos Inc	North America	P/C insurer	x	x	x
L	Loews Corp	North America	P/C insurer	x	x	x
MCY	Mercury General Corp	North America	P/C insurer	x	x	x
MKL	Markel Corp	North America	P/C insurer	x	x	x
PGR	Progressive Corp	North America	P/C insurer	x	x	x
PRA	ProAssurance Corp	North America	P/C insurer	x	x	x
PTVCB	Protective Insurance Corp	North America	P/C insurer	x	x	x
RE	Everest Re Group Ltd	North America	P/C insurer	x	x	x
SAFT	Safety Insurance Group Inc	North America	P/C insurer		x	x
SBBG	Seibels Bruce Group Inc	North America	P/C insurer	x	x	x
SIGI	Selective Insurance Group Inc	North America	P/C insurer	x	x	x
STFC	State Auto Financial Corp	North America	P/C insurer	x	x	x
THG	Hanover Insurance Group Inc	North America	P/C insurer	x	x	x
TRV	Travelers Cos Inc	North America	P/C insurer	x	x	x
UIHC	United Insurance Holdings Corp	North America	P/C insurer			x
UNAM	Unico American Corp	North America	P/C insurer	x	x	x
UVE	Universal Insurance Holdings Inc	North America	P/C insurer	x	x	x
WRB	WR Berkley Corp	North America	P/C insurer	x	x	x
WTM	White Mountains Insurance Group Ltd	North America	P/C insurer	x	x	x
Y	Alleghany Corp	North America	P/C insurer	x	x	x
CB	Chubb Ltd	Europe	P/C insurer	x	x	x

Table 19: Companies in the dataset I.

Ticker	Company name	Location	Sector	Pre-crisis	Crisis	Post-crisis
AEL	American Equity Investment Life Holding Co	North America	Life insurer		x	x
CIA	Citizens Inc	North America	Life insurer	x	x	x
FFG	FBL Financial Group Inc	North America	Life insurer	x	x	x
GL	Globe Life Inc	North America	Life insurer	x	x	x
GNW	Genworth Financial Inc	North America	Life insurer		x	x
GWO	Great-West Lifeco Inc	North America	Life insurer	x	x	x
IHC	Independence Holding Co	North America	Life insurer	x	x	x
KCLI	Kansas City Life Insurance Co	North America	Life insurer	x	x	x
LNC	Lincoln National Corp	North America	Life insurer	x	x	x
MET	MetLife Inc	North America	Life insurer	x	x	x
NWLI	National Western Life Group Inc	North America	Life insurer	x	x	x
POW	Power Corp of Canada	North America	Life insurer	x	x	x
PRU	Primerica Inc	North America	Life insurer		x	x
PWF	Power Financial Corp	North America	Life insurer	x	x	x
UTGN	UTG Inc	North America	Life insurer	x	x	x
AGN	Aegon NV	Europe	Life insurer		x	x
AV/	Aviva PLC	Europe	Life insurer	x	x	x
PRU LN	Prudential PLC	Europe	Life insurer	x	x	x
AFL	Aflac Inc	North America	Accident and health insurer	x	x	x
AIZ	Assurant Inc	North America	Accident and health insurer		x	x
CFIN	Citizens Financial Corp	North America	Accident and health insurer	x	x	x
CNO	CNO Financial Group Inc	North America	Accident and health insurer		x	x
MFC	Manulife Financial Corp	North America	Accident and health insurer	x	x	x
PFG	Principal Financial Group Inc	North America	Accident and health insurer	x	x	x
SLF	Sun Life Financial Inc	North America	Accident and health insurer	x	x	x
UNM	Unum Group	North America	Accident and health insurer	x	x	x

Table 20: Companies in the dataset II.

Ticker	Company name	Location	Sector	Pre-crisis	Crisis	Post-crisis
AGO	Assured Guaranty Ltd	North America	Surety and title insurer		x	x
ARGO	Argo Group International Holdings Ltd	North America	Surety and title insurer	x	x	x
ITIC	Investors Title Co	North America	Surety and title insurer	x	x	x
MBI	MBIA Inc	North America	Surety and title insurer	x	x	x
MTG	MGIC Investment Corp	North America	Surety and title insurer	x	x	x
ORI	Old Republic International Corp	North America	Surety and title insurer	x	x	x
RDN	Radian Group Inc	North America	Surety and title insurer	x	x	x
STC	Stewart Information Services Corp	North America	Surety and title insurer	x	x	x
SYCRF	Syncora Holdings Ltd	North America	Surety and title insurer		x	x
TGIC	Triad Guaranty Inc	North America	Surety and title insurer	x	x	x
AOREF	American Overseas Group Ltd	North America	Reinsurer		x	x
BRK/A	Berkshire Hathaway Inc	North America	Reinsurer	x	x	x
ESGR	Enstar Group Ltd	North America	Reinsurer		x	x
MHLD	Maiden Holdings Ltd	North America	Reinsurer			x
RGA	Reinsurance Group of America Inc	North America	Reinsurer	x	x	x
RNR	RenaissanceRe Holdings Ltd	North America	Reinsurer	x	x	x
RQIH	Randall and Quilter Investment Holdings Ltd	North America	Reinsurer			x
DNREM	Dunav Re a.d	Europe	Reinsurer			x
HNR1	Hannover Rueck SE	Europe	Reinsurer	x	x	x
MUV2	Muenchener Rueckversicherungs-Gesellschaft AG in Muenchen	Europe	Reinsurer	x	x	x
SCR	SCOR SE	Europe	Reinsurer		x	x
SREN	Swiss Re AG	Europe	Reinsurer	x	x	x

Table 21: Companies in the dataset III.

Ticker	Company name	Location	Sector	Pre-crisis	Crisis	Post-crisis
AIG	American International Group Inc	North America	Full line insurer	x	x	x
ANAT	American National Insurance Co	North America	Full line insurer	x	x	x
CNA	CNA Financial Corp	North America	Full line insurer	x	x	x
ELF	E-L Financial Corp Ltd	North America	Full line insurer	x	x	x
HIG	Hartford Financial Services Group Inc	North America	Full line insurer	x	x	x
KMPR	Kemper Corp	North America	Full line insurer	x	x	x
NSEC	National Security Group Inc	North America	Full line insurer	x	x	x
SGYI	Strategy International Insurance Group Inc	North America	Full line insurer		x	x
UFCS	United Fire Group Inc	North America	Full line insurer	x	x	x
ALMB	Alm Brand A/S	Europe	Full line insurer		x	x
ALV	Allianz SE	Europe	Full line insurer	x	x	x
BALN	Baloise Holding AG	Europe	Full line insurer	x	x	x
CS	AXA SA	Europe	Full line insurer		x	x
FBD	FBD Holdings PLC	Europe	Full line insurer		x	x
G	Assicurazioni Generali SpA	Europe	Full line insurer	x	x	x
HELN	Helvetia Holding AG	Europe	Full line insurer	x	x	x
MAP	Mapfre SA	Europe	Full line insurer		x	x
MINE	Minerva Insurance Co Public Ltd	Europe	Full line insurer	x	x	x
MMS	MAXIMUS Inc	Europe	Full line insurer			x
NBG6	NUERNBERGER Beteiligungs AG	Europe	Full line insurer		x	x
PROTCT	Protector Forsikring ASA	Europe	Full line insurer		x	x
RAYSG	Ray Sigorta AS	Europe	Full line insurer		x	x
RSA	RSA Insurance Group PLC	Europe	Full line insurer	x	x	x
STB	Storebrand ASA	Europe	Full line insurer	x	x	x
TRYG	Tryg A/S	Europe	Full line insurer		x	x
UNI	Unipol Gruppo SpA	Europe	Full line insurer	x	x	x
UQA	UNIQA Insurance Group AG	Europe	Full line insurer		x	x
VAHN	Vaudoise Assurances Holding SA	Europe	Full line insurer		x	x
VIG	Vienna Insurance Group AG Wiener Versicherung	Europe	Full line insurer		x	x
ZURN	Zurich Insurance Group AG	Europe	Full line insurer	x	x	x

Table 22: Companies in the dataset IV.

Ticker	Company name	Location	Sector	Pre-crisis	Crisis	Post-crisis
BAC	Bank of America Corp	North America	Bank	x	x	x
BMO	Bank of Montreal	North America	Bank	x	x	x
BK	Bank of New York Mellon Corp	North America	Bank	x	x	x
BNS	Bank of Nova Scotia	North America	Bank	x	x	x
CM	Canadian Imperial Bank of Commerce	North America	Bank	x	x	x
COF	Capital One Financial Corp	North America	Bank	x	x	x
C	Citigroup Inc	North America	Bank	x	x	x
JPM	JPMorgan Chase and Co	North America	Bank	x	x	x
PNC	PNC Financial Services Group Inc	North America	Bank	x	x	x
RY	Royal Bank of Canada	North America	Bank	x	x	x
STT	State Street Corp	North America	Bank	x	x	x
TD	Toronto-Dominion Bank	North America	Bank	x	x	x
TFC	Truist Financial Corp	North America	Bank	x	x	x
USB	US Bancorp	North America	Bank	x	x	x
WFC	Wells Fargo and Co	North America	Bank	x	x	x
BHBCQ	Beverly Hills Bancorp Inc	North America	Bank	x	x	x
GABA	Georgia Bancshares Inc	North America	Bank		x	x
HABK	Hamilton Bancorp Inc	North America	Bank	x	x	x
NXTYQ	Nexity Financial Corp	North America	Bank		x	x
RPFG	Rainier Pacific Financial Group Inc	North America	Bank		x	x
SFBI	SFSB Inc	North America	Bank		x	x
TMCV	Temecula Valley Bancorp Inc	North America	Bank		x	x
WSFGQ	WSB Financial Group Inc	North America	Bank		x	x
ACA	Credit Agricole SA	Europe	Bank		x	x
BARC	Barclays PLC	Europe	Bank	x	x	x
BBVA	Banco Bilbao Vizcaya Argentaria SA	Europe	Bank	x	x	x
BNP	BNP Paribas SA	Europe	Bank			x
CSGN	Credit Suisse Group AG	Europe	Bank	x	x	x
DBK	Deutsche Bank AG	Europe	Bank	x	x	x
DNB	DNB ASA	Europe	Bank	x	x	x
GLE	Societe Generale SA	Europe	Bank		x	x
HSBA	HSBC Holdings PLC	Europe	Bank		x	x
INGA	ING Groep NV	Europe	Bank	x	x	x

Table 23: Companies in the dataset V.

Ticker	Company name	Location	Sector	Pre-crisis	Crisis	Post-crisis
ISP	Intesa Sanpaolo SpA	Europe	Bank	x	x	x
KBC	KBC Group NV	Europe	Bank		x	x
LLOY	Lloyds Banking Group PLC	Europe	Bank		x	x
RBS	Royal Bank of Scotland Group PLC	Europe	Bank		x	x
SAN	Banco Santander SA	Europe	Bank	x	x	x
SEBA	Skandinaviska Enskilda Banken AB	Europe	Bank	x	x	x
SHBA	Svenska Handelsbanken AB	Europe	Bank	x	x	x
STAN	Standard Chartered PLC	Europe	Bank	x	x	x
SWEDA	Swedbank AB	Europe	Bank	x	x	x
UBSG	UBS Group AG	Europe	Bank	x	x	x
UCG	UniCredit SpA	Europe	Bank	x	x	x
GARAN	Turkiye Garanti Bankasi AS	Europe	Bank	x	x	x
ING	ING BANK ŚLĄSKI	Europe	Bank	x	x	x
PEO	Bank Polska Kasa Opieki SA	Europe	Bank	x	x	x
PKO	Pko Bank Polski SA	Europe	Bank		x	x
QNBFB	QNB Finansbank AS	Europe	Bank	x	x	x
SPL	Santander Bank Polska SA	Europe	Bank	x	x	x

Table 24: Companies in the dataset VI.

Sector	North America	Europe	Pre-crisis	Crisis	Post-crisis
P/C insurance	x		25	30	32
		x	1	1	1
Life insurance	x		12	15	15
		x	2	3	3
Accident and health insurance	x		6	8	8
Surety and title insurance	x		8	10	10
Reinsurance	x		3	5	7
		x	3	4	5
Full line insurance	x		8	9	9
		x	9	20	21
Big bank	x		15	15	15
		x	14	20	21
Small bank	x		2	8	8
		x	5	6	6

Table 25: Sectorial aggregation of companies in different locations and period

B Examples

B.1 Remarks to the examples

Five examples will be discussed related to networks ($m = 1,2,3,4,5$). Using consistent formalism during the discussion, I declare some common signs I will use.

- Y_t^m describes the multivariate time series matrix in the m^{th} case reflecting its values on time t.

  $$Y^{(m)} = \begin{bmatrix} y_1^{(m)} \\ y_2^{(m)} \\ y_3^{(m)} \end{bmatrix}$$

  , where ${}_i^{(m)}$ $i = 1,2,3,(4,5)$ signs the whole time series.

- $A^{(m)}$ is the coefficient matrix in the m^{th} case.

- $u_t^{(m)}$ is the matrix of innovations in the m^{th} example on time t. Where $u_{k,t}^{(m)}$ $k = 1,2,3$ follows standard normal distribution.

- The $\alpha_i^{(m)}$, $i = 1,2$ coefficients expresses the contribution of the own first and second lags to the dependent variable, while $\gamma_j^{(m)}$, $j = 1,2$ provides information for the effect of first and second lagged values of the dependent variable. E.g. $y_1^{(m)}$ is explained by its own lags and $y_2^{(m)}$, then the α_i coefficients belongs to $y_{1,t}^{(m)}$, while $\gamma_j^{(m)}$ values to the $y_2^{(m)}$ time series.

B.2 Example I. of Hué et al. (2019)

$$\underbrace{\begin{bmatrix} y_{1,t}^{(1)} \\ y_{2,t}^{(1)} \\ y_{3,t}^{(1)} \end{bmatrix}}_{Y_t^{(1)}} = \underbrace{\begin{bmatrix} 0.5 & 0 & 0 \\ 0.2 & 0.5 & 0 \\ 0.2 & 0 & 0.5 \end{bmatrix}}_{A^{(1)}} \cdot \underbrace{\begin{bmatrix} y_{1,t-1}^{(1)} \\ y_{2,t-1}^{(1)} \\ y_{3,t-1}^{(1)} \end{bmatrix}}_{Y_{t-1}^{(1)}} + \underbrace{\begin{bmatrix} u_{1,t}^{(1)} \\ u_{2,t}^{(1)} \\ u_{3,t}^{(1)} \end{bmatrix}}_{u_t^{(1)}} \tag{60}$$

The structure of the network implies that institution 1 Granger-causes institution 2 and institution 3, and a spurious connection appears between institution 2 and institution 3.

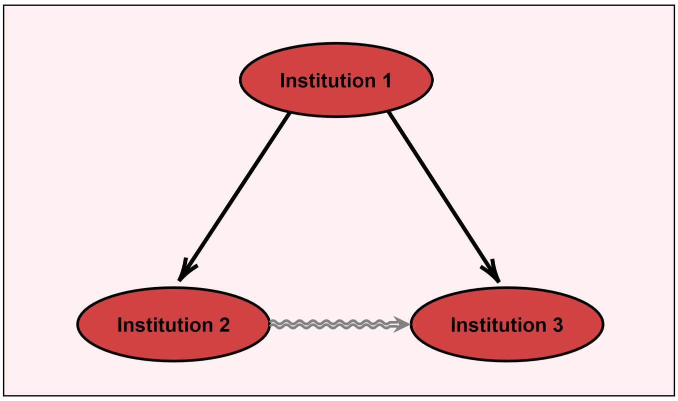

Figure 17: Network graph I. of Hué et al. (2019)

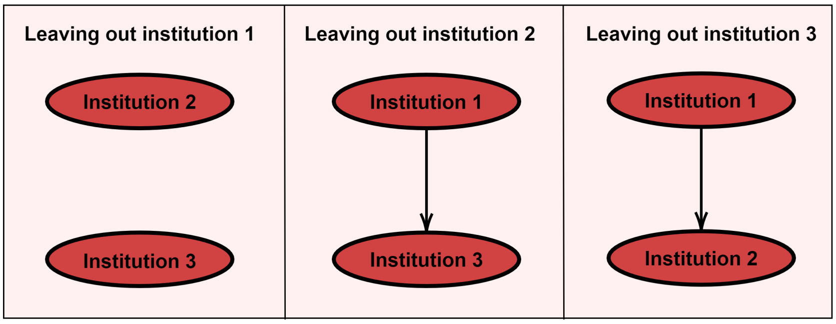

Figure 18: Leave-one-out relations I. of Hué et al. (2019)

B.3 Example II. of Hué et al. (2019)

$$\underbrace{\begin{bmatrix} y_{1,t}^{(2)} \\ y_{2,t}^{(2)} \\ y_{3,t}^{(2)} \end{bmatrix}}_{Y_t^{(2)}} = \underbrace{\begin{bmatrix} 0.5 & 0 & 0 \\ 0.2 & 0.5 & 0 \\ 0 & 0.2 & 0.5 \end{bmatrix}}_{A^{(2)}} \cdot \underbrace{\begin{bmatrix} y_{1,t-1}^{(2)} \\ y_{2,t-1}^{(2)} \\ y_{3,t-1}^{(2)} \end{bmatrix}}_{Y_{t-1}^{(2)}} + \underbrace{\begin{bmatrix} u_{1,t}^{(2)} \\ u_{2,t}^{(2)} \\ u_{3,t}^{(2)} \end{bmatrix}}_{u_t^{(2)}} \tag{61}$$

The network graph is designed as follows: institution 1 Granger-cause institution 2, institution 2 has an effect on institution 3. Last but not least, a side-effect arises thanks to the described linkages a spurious causality from institution 1 to institution 3.

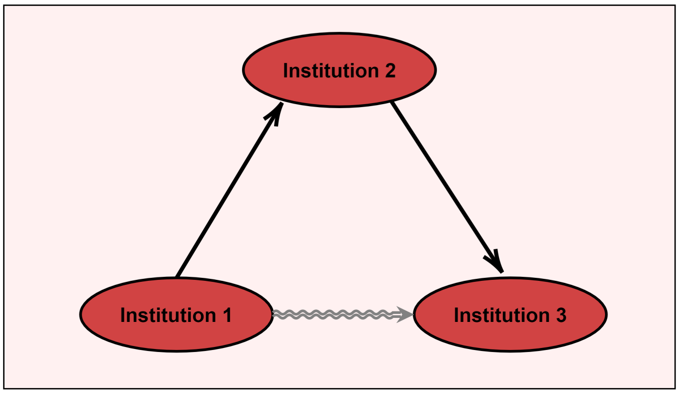

Figure 19: Network graph II. of Hué et al. (2019)

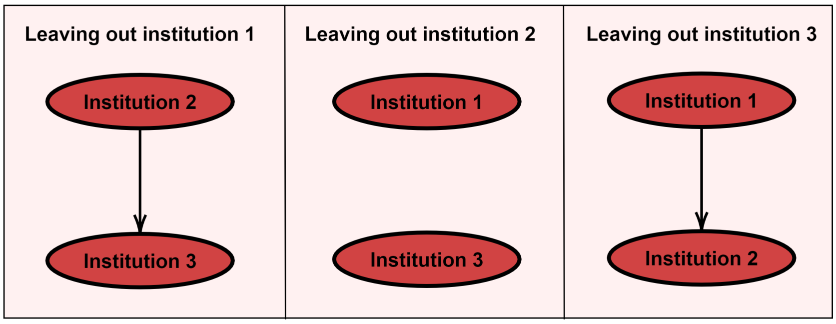

Figure 20: Leave-one-out relations II. of Hué et al. (2019)

B.4 Modification of example I. (Hué et al. 2019)

$$\underbrace{\begin{bmatrix} y_{1,t}^{(3)} \\ y_{2,t}^{(3)} \\ y_{3,t}^{(3)} \\ y_{4,t}^{(3)} \\ y_{5,t}^{(3)} \end{bmatrix}}_{Y_t^{(3)}} = \underbrace{\begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 \\ 0.2 & 0.5 & 0 & 0 & 0 \\ 0.2 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.2 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}}_{A^{(3)}} \cdot \underbrace{\begin{bmatrix} y_{1,t-1}^{(3)} \\ y_{2,t-1}^{(3)} \\ y_{3,t-1}^{(3)} \\ y_{4,t-1}^{(3)} \\ y_{5,t-1}^{(3)} \end{bmatrix}}_{Y_{t-1}^{(3)}} + \underbrace{\begin{bmatrix} u_{1,t}^{(3)} \\ u_{2,t}^{(3)} \\ u_{3,t}^{(3)} \\ u_{4,t}^{(3)} \\ u_{5,t}^{(3)} \end{bmatrix}}_{u_t^{(3)}} \tag{62}$$

The structure of the network implies that institution 1 Granger-causes institution 2 and institution 3, and a spurious connection appears between institution 2 and institution 3. Furthermore, institution 3 Granger-causes institution 4, but institution 5 is an isolated node, which means that this node has effects only on itself.

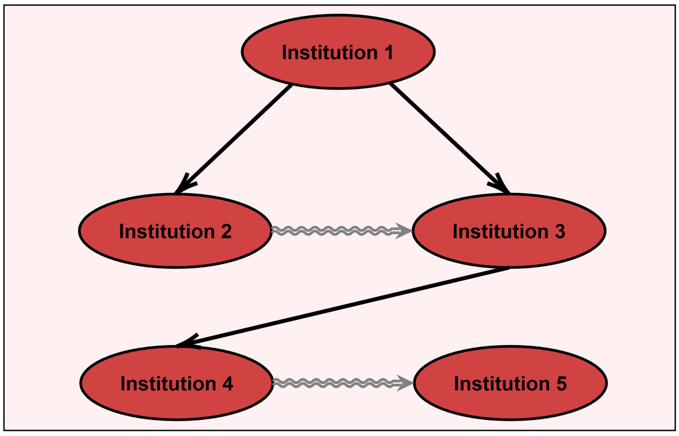

Figure 21: Modified network graph of the example I. from Hué et al. (2019)

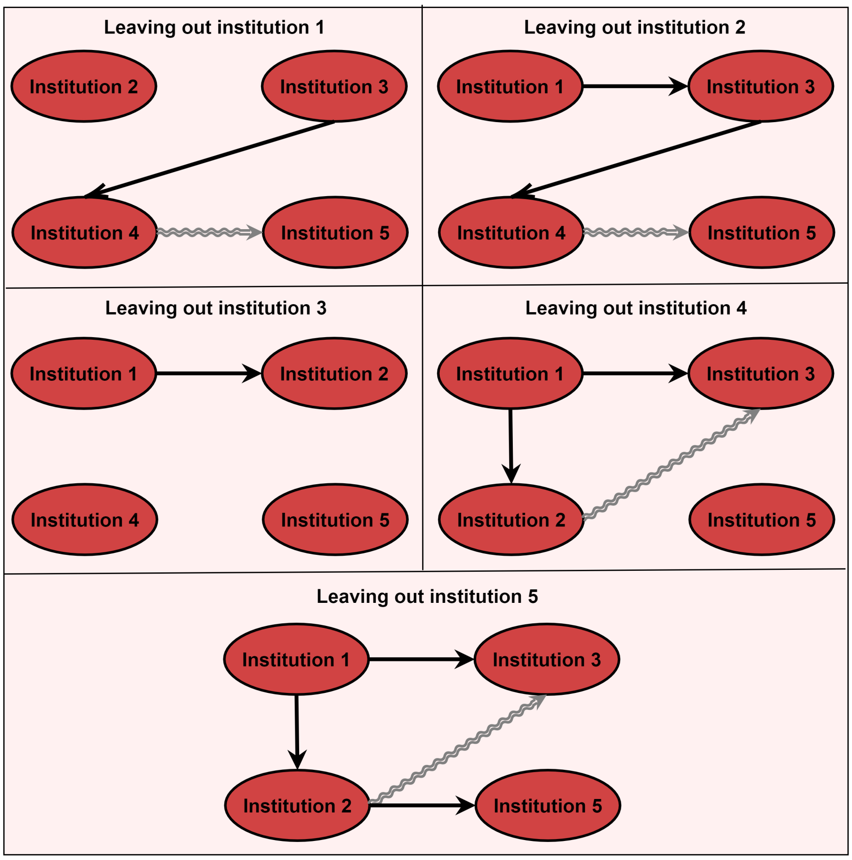

Figure 22: Leave-one-out relations of the example I. from Hué et al. (2019)

B.5 Example IV. based on the idea of Song and Taamouti (2019)

$$\underbrace{\begin{bmatrix} y^{(4)}_{1,t} \\ y^{(4)}_{2,t} \\ y^{(4)}_{3,t} \end{bmatrix}}_{Y^{(4)}_t} = \underbrace{\begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 & 0 \\ -0.2 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0.1 & 0 & 0 \end{bmatrix}}_{A^{(4)}} \cdot \underbrace{\begin{bmatrix} y^{(4)}_{1,t-1} \\ y^{(4)}_{2,t-1} \\ y^{(4)}_{3,t-1} \\ y^{(4)}_{1,t-2} \\ y^{(4)}_{2,t-2} \\ y^{(4)}_{3,t-2} \end{bmatrix}}_{Y^{(4)}_{t-1}} + \underbrace{\begin{bmatrix} u^{(4)}_{1,t} \\ u^{(4)}_{2,t} \\ u^{(4)}_{3,t} \end{bmatrix}}_{u^{(4)}_t} \tag{63}$$

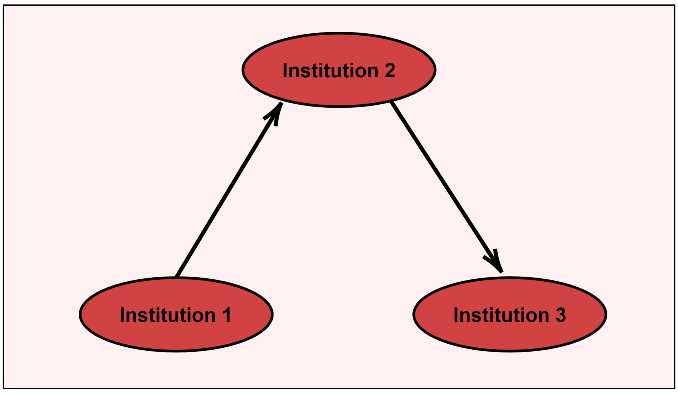

Figure 23: Network graph modified example of Song and Taamouti (2019)

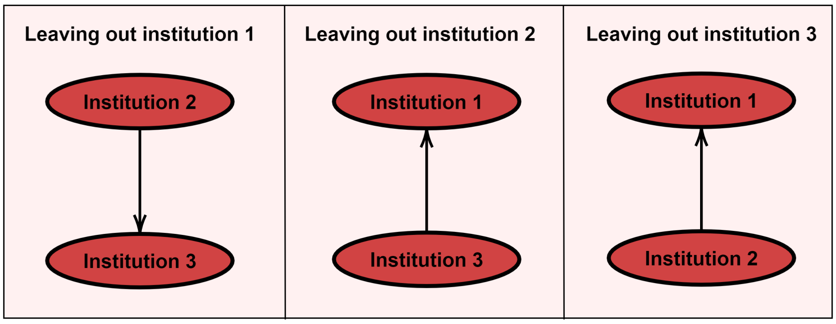

Figure 24: Leave-one-out relations of the modified example of Song and Taamouti (2019)

This example is unique, while it demonstrates the case when leaving out a node will reveal a new connection in the network.

In this network, institution 1 Granger-causes institution 2, and institution 2 has an effect on institution 3. Although seemingly institution 1 and institution 3 are related, they are not, while:

$$
\begin{aligned}
y_{3,t}^{(4)} =& 0.5 y_{3,t-1}^{(4)} + 0.1 y_{1,t-2} + 0.5 y_{2,t-1}^{(4)} + u_{3,t}^{(4)} = \\
& 0.5 y_{3,t-1}^{(4)} + 0.1 y_{1,t-2} + 0.5(0.5 y_{2,t-2} - 0.2 y_{1,t-2} + u_{2,t-1}) + u_{3,t}^{(4)} = \\
& 0.5 y_{3,t-1}^{(4)} + 0.1 y_{1,t-2} + 0.25 y_{2,t-2} - 0.1 y_{1,t-2} + 0.5 u_{2,t-1} + u_{3,t}^{(4)} = \\
& 0.5 y_{3,t-1}^{(4)} + 0.25 y_{2,t-2} + 0.5 u_{2,t-1} + u_{3,t}^{(4)}
\end{aligned} \tag{64}
$$

Excluding the 2^{nd} institution from the network will create a new edge between institution 1 and institution 3.

B.6 Example V. (Feizi et al. 2013)

$$
\underbrace{\begin{bmatrix} y_{1,t}^{(5)} \\ y_{2,t}^{(5)} \\ y_{3,t}^{(5)} \\ y_{4,t}^{(5)} \\ y_{5,t}^{(5)} \end{bmatrix}}_{Y_t^{(5)}} = \underbrace{\begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 \\ 0.2 & 0.5 & 0 & 0 & 0 \\ 0 & 0.2 & 0.5 & 0 & 0 \\ 0 & 0.2 & 0.2 & 0.5 & 0 \\ 0 & 0 & 0.2 & 0.2 & 0.5 \end{bmatrix}}_{A^{(5)}} \cdot \underbrace{\begin{bmatrix} y_{1,t-1}^{(5)} \\ y_{2,t-1}^{(5)} \\ y_{3,t-1}^{(5)} \\ y_{4,t-1}^{(5)} \\ y_{5,t-1}^{(5)} \end{bmatrix}}_{Y_{t-1}^{(5)}} + \underbrace{\begin{bmatrix} u_{1,t}^{(5)} \\ u_{2,t}^{(5)} \\ u_{3,t}^{(5)} \\ u_{4,t}^{(5)} \\ u_{5,t}^{(5)} \end{bmatrix}}_{u_t^{(5)}} \tag{65}
$$

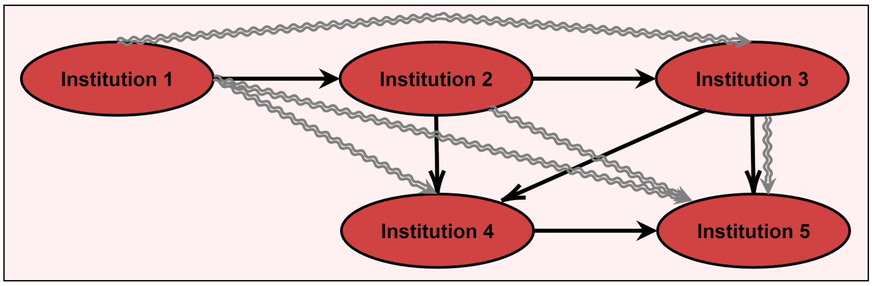

Figure 25: Network graph of Feizi et al. (2013)

The network is quite complex. There are direct connections between institution 1 and institution 2, institution 2 and institution 3, institution 2 and institution 4, institution 3 and institution 5 and institution 4 and institution 5. Further spurious edges rise from node 1 to node 3, node 4 and node 5. Another indirect relation exists from node 2 to node 5 and from node 3 to node 5.

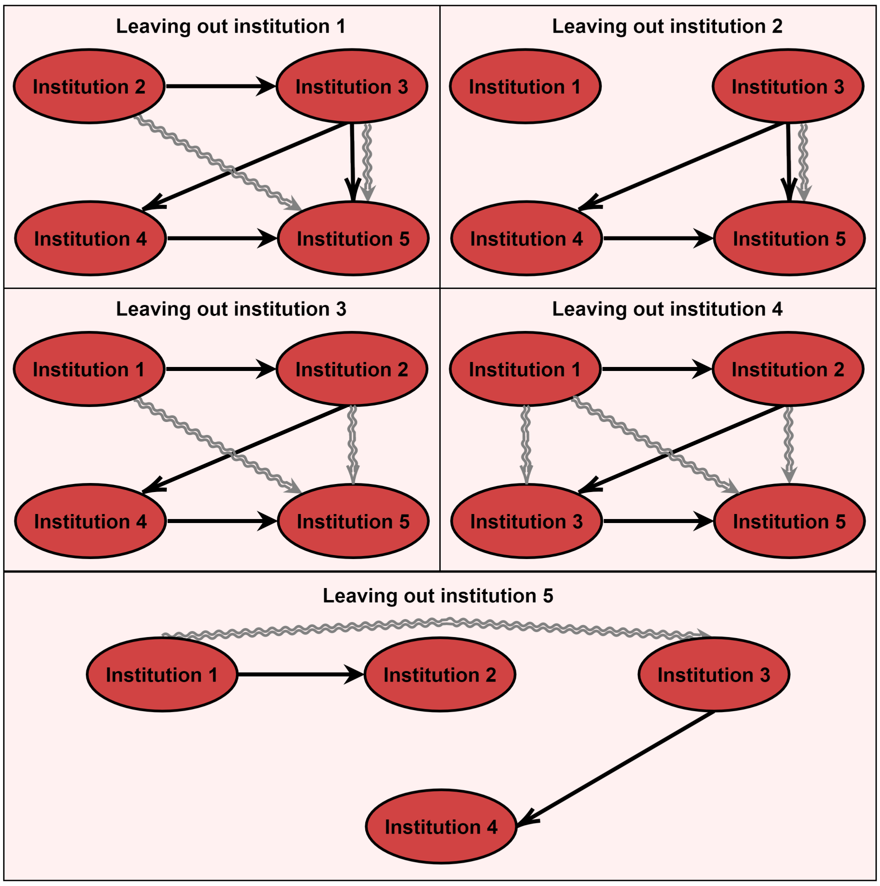

Figure 26: Leave-one-out relations of Feizi et al. (2013)

C Methodological supplement

C.1 Testing indirect and spurious effects

Variable/Statistics	Value
τ_1^{Ind}	$(\eta, \gamma^T, \lambda^T, \theta^T)^T$
z_{1t}	$(1, X_t, ..., X_{t+1-\overline{p}}, Y_t, ..., Y_{t+1-\overline{q}}, f_t, ..., f_{t+1-\overline{h}})$
$R^{Ind,\lambda}$	$(0_{\overline{q},1+\overline{p}}, I_{\overline{q}}, 0_{\overline{q},\overline{h}})$
ϵ_{t+1}	$X_{t+1} - z_{1t}\tau_1^{Ind}$
$\Sigma_{\tau_1^{Ind}}$	$\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{1t}z_{1t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} \epsilon_{t+1}^2 z_{1t}z_{1t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{1t}z_{1t}^T\right)^{-1}$
$F_T^{Ind,\lambda}$	$\left(\sqrt{T}(\tau_1^{Ind})^T(R^{Ind,\lambda})^T\right)\left(R^{Ind,\lambda}\Sigma_{\tau_1^{Ind}}(R^{Ind,\lambda})^T\right)^{-1}\left(\sqrt{T}(\tau_1^{Ind})^T(R^{Ind,\lambda})^T\right)^T \sim \chi^2_{\overline{q}}$

Source: Song and Taamouti (2019), p.923-924

Table 26: Parameters for testing indirect causality condition 2

Variable/Statistics	Value
τ_2^{Ind}	$(\nu, \kappa^T, \psi^T, \rho^T)^T$
z_{2t}	$(1, X_t, ..., X_{t+1-\dot{p}}, Y_t, ..., Y_{t+1-\dot{q}}, f_t, ..., f_{t+1-\dot{h}})$
$R^{Ind,\psi}$	$(0_{\dot{q},1+\dot{p}}, I_{\dot{q}}, 0_{\dot{q},\dot{h}})$
u_{t+1}	$f_{t+1} - z_{2t}\tau_2^{Ind}$
$\Sigma_{\tau_2^{Ind}}$	$\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{2t}z_{2t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} u_{t+1}^2 z_{2t}z_{2t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{2t}z_{2t}^T\right)^{-1}$
$F_T^{Ind,\psi}$	$\left(\sqrt{T}(\tau_2^{Ind})^T(R^{Ind,\psi})^T\right)\left(R^{Ind,\psi}\Sigma_{\tau_2^{Ind}}(R^{Ind,\psi})^T\right)^{-1}\left(\sqrt{T}(\tau_2^{Ind})^T(R^{Ind,\psi})^T\right)^T \sim \chi^2_{\dot{q}}$
τ_3^{Ind}	$(\overline{\omega}, \xi^T, \delta^T, \zeta^T)^T$
z_{3t}	$(1, X_t, ..., X_{t+1-\ddot{p}}, Y_t, ..., Y_{t+1-\ddot{q}}, f_t, ..., f_{t+1-\ddot{h}})$
$R^{Ind,\zeta}$	$(0_{\ddot{h},1+\ddot{p}+\ddot{q}}, I_{\ddot{h}})$
u_{t+1}	$f_{t+1} - z_{3t}\tau_3^{Ind}$
$\Sigma_{\tau_3^{Ind}}$	$\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{3t}z_{3t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} \epsilon_{t+1}^3 z_{3t}z_{3t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{3t}z_{3t}^T\right)^{-1}$
$F_T^{Ind,\zeta}$	$\left(\sqrt{T}(\tau_3^{Ind})^T(R^{Ind,\zeta})^T\right)\left(R^{Ind,\zeta}\Sigma_{\tau_3^{Ind}}(R^{Ind,\zeta})^T\right)^{-1}\left(\sqrt{T}(\tau_3^{Ind})^T(R^{Ind,\zeta})^T\right)^T \sim \chi^2_{\ddot{h}}$

Source: Song and Taamouti (2019), p.924-925

Table 27: Parameters for testing indirect causality condition 3

Variable/Statistics	Value
τ_1^{si}	$(\mu, \beta^T, \alpha^T, \pi^T)^T$
z_{1t}^{si}	$(1, X_t, ..., X_{t+1-\overline{p}}, Y_t, ..., Y_{t+1-\overline{q}}, f_t, ..., f_{t+1-\overline{h}})$
$R^{si,\alpha}$	$(0_{\overline{q},1+\overline{p}}, I_{\overline{q}}, 0_{\overline{q},\overline{h}})$
ϵ_{t+1}	$X_{t+1} - z_{1t}^{si}\tau_1^{si}$
$\Sigma_{\tau_1^{si}}$	$\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{1t}^{si}(z_{1t}^{si})^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} \epsilon_{t+1}^2 z_{1t}^{si}(z_{1t}^{si})^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{1t}^{si}(z_{1t}^{si})^T\right)^{-1}$
$F_T^{si,\alpha}$	$\left(\sqrt{T}(\tau_1^{si})^T(R^{si,\alpha})^T\right)\left(R^{si,\alpha}\Sigma_{\tau_1^{si}}(R^{si,\alpha})^T\right)^{-1}\left(\sqrt{T}(\tau_1^{si})^T(R^{si,\alpha})^T\right)^T \sim \chi_{\overline{q}}^2$

Source: Song and Taamouti (2019), p.925-926

Table 28: Parameters for testing spurious causality type 1 condition 1

Variable/Statistics	Value
τ_2^{si}	$(\nu, \kappa^T, \psi^T, \rho^T)^T$
z_{2t}	$(1, X_t, ..., X_{t+1-\dot{p}}, Y_t, ..., Y_{t+1-\dot{q}}, f_t, ..., f_{t+1-\dot{h}})$
$R^{si,\psi}$	$(0_{\dot{q},1+\dot{p}}, I_{\dot{q}}, 0_{\dot{q},\dot{h}})$
u_{t+1}	$f_{t+1} - z_{2t}\tau_2^{si}$
$\Sigma_{\tau_2^{si}}$	$\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{2t}z_{2t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} u_{t+1}^2 z_{2t}z_{2t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{2t}z_{2t}^T\right)^{-1}$
$F_T^{si,\psi}$	$\left(\sqrt{T}(\tau_2^{si})^T(R^{si,\psi})^T\right)\left(R^{si,\psi}\Sigma_{\tau_2^{si}}(R^{si,\psi})^T\right)^{-1}\left(\sqrt{T}(\tau_2^{si})^T(R^{si,\psi})^T\right)^T \sim \chi_{\dot{q}}^2$
τ_3^{si}	$(\overline{\omega}, \xi^T, \delta^T, \zeta^T)^T$
z_{3t}	$(1, X_t, ..., X_{t+1-\ddot{p}}, Y_t, ..., Y_{t+1-\ddot{q}}, f_t, ..., f_{t+1-\ddot{h}})$
$R^{si,\zeta}$	$(0_{\ddot{h},1+\ddot{p}}, I_{\ddot{q}}, 0_{\ddot{q},\ddot{h}})$
ϵ_{t+1}	$f_{t+1} - z_{3t}\tau_3^{si}$
$\Sigma_{\tau_3^{si}}$	$\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{3t}z_{3t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} \epsilon_{t+1}^3 z_{3t}z_{3t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{3t}z_{3t}^T\right)^{-1}$
$F_T^{si,\zeta}$	$\left(\sqrt{T}(\tau_3^{si})^T(R^{si,\zeta})^T\right)\left(R^{si,\zeta}\Sigma_{\tau_3^{si}}(R^{si,\zeta})^T\right)^{-1}\left(\sqrt{T}(\tau_3^{si})^T(R^{si,\zeta})^T\right)^T \sim \chi_{\ddot{h}}^2$

Source: Song and Taamouti (2019), p.925-926

Table 29: Parameters for testing spurious causality type 1 condition 3

Variable/Statistics	Value
τ_1^{sii}	$(\eta, \gamma^T, \lambda^T, \theta^T)^T$
z_{1t}^{sii}	$(1, X_t, ..., X_{t+1-\overline{p}}, Y_t, ..., Y_{t+1-\overline{q}}, f_t, ..., f_{t+1-\overline{h}})$
$R^{sii,\lambda}$	$(0_{\overline{q},1+\overline{p}}, I_{\overline{q}}, 0_{\overline{q},\overline{h}})$
ϵ_{t+1}	$X_{t+1} - z_{1t}^{sii}\tau_1^{sii}$
$\Sigma_{\tau_1^{sii}}$	$\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{1t}^{sii}(z_{1t}^{sii})^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} \epsilon_{t+1}^2 z_{1t}^{sii}(z_{1t}^{sii})^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{1t}^{sii}(z_{1t}^{sii})^T\right)^{-1}$
$F_T^{sii,\lambda}$	$\left(\sqrt{T}(\tau_1^{sii})^T(R^{sii,\lambda})^T\right)\left(R^{sii,\lambda}\Sigma_{\tau_1^{sii}}(R^{sii,\lambda})^T\right)^{-1}\left(\sqrt{T}(\tau_1^{sii})^T(R^{sii,\lambda})^T\right)^T \sim \chi_{\overline{q}}^2$

Source: Song and Taamouti (2019), p.926-927

Table 30: Parameters for testing spurious causality type 2 condition 2

Variable/Statistics	Value
τ_2^{sii}	$(\nu, \kappa^T, \psi^T, \rho^T)^T$
z_{2t}	$(1, X_t, ..., X_{t+1-\dot{p}}, Y_t, ..., Y_{t+1-\dot{q}}, f_t, ..., f_{t+1-\dot{h}})$
$R^{sii,\psi}$	$(0_{\dot{h},1+\dot{p}+\dot{q}}, I_{\dot{h}})$
u_{t+1}	$Y_{t+1} - z_{2t}\tau_2^{sii}$
$\Sigma_{\tau_2^{sii}}$	$\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{2t}z_{2t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} u_{t+1}^2 z_{2t}z_{2t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{2t}z_{2t}^T\right)^{-1}$
$F_T^{sii,\psi}$	$\left(\sqrt{T}(\tau_2^{sii})^T(R^{sii,\psi})^T\right)\left(R^{sii,\psi}\Sigma_{\tau_2^{sii}}(R^{sii,\psi})^T\right)^{-1}\left(\sqrt{T}(\tau_2^{sii})^T(R^{sii,\psi})^T\right)^T \sim \chi_{\dot{h}}^2$
τ_3^{sii}	$(\overline{\omega}, \xi^T, \delta^T, \zeta^T)^T$
z_{3t}	$(1, X_t, ..., X_{t+1-\ddot{p}}, Y_t, ..., Y_{t+1-\ddot{q}}, f_t, ..., f_{t+1-\ddot{h}})$
$R^{sii,\zeta}$	$(0_{\ddot{h},1+\ddot{p}}, I_{\ddot{q}}, 0_{\ddot{q},\ddot{h}})$
ϵ_{t+1}	$f_{t+1} - z_{3t}\tau_3^{sii}$
$\Sigma_{\tau_3^{sii}}$	$\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{3t}z_{3t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} \epsilon_{t+1}^3 z_{3t}z_{3t}^T\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T-1} z_{3t}z_{3t}^T\right)^{-1}$
$F_T^{sii,\zeta}$	$\left(\sqrt{T}(\tau_3^{sii})^T(R^{sii,\zeta})^T\right)\left(R^{sii,\zeta}\Sigma_{\tau_3^{sii}}(R^{sii,\zeta})^T\right)^{-1}\left(\sqrt{T}(\tau_3^{sii})^T(R^{sii,\zeta})^T\right)^T \sim \chi_{\ddot{h}}^2$

Source: Song and Taamouti (2019), p.926-927

Table 31: Parameters for testing spurious causality type 2 condition 3

C.2 Monte Carlo simulation

I applied the Monte Carlo simulation method to compare the efficiency of the methods proposed by Hué et al. (2019) and Song and Taamouti (2019). A simulated two examples **??** and B.6. I tried to detect Granger-causality transferred by non-zero coefficients, and I created table 32 to compare how many times was identified significant relationships among nodes. I repeated the causality test in the first case 1000 times and in the latter case 500 times. I run the simulation for sample sizes $T = 100, 250, 500, 1000, 3000, 5000$. After that, I calculated the average hit ratios for every non-zero parameter in all cases.

The table shows clearly that for small samples ($T = 100, 250$), the former method admitted more precise, but in medium sample size ($T = 500$), the outcome is very close. For larger samples, you cannot find any difference in the hit ratio.

The simulation concluded that for medium and large samples are only adviced to use the framework of Song and Taamouti (2019). Nevertheless, I remark, in this particular case, the principal component was calculated only from one variable, so the exercise was too small to enjoy the advantages of the PCA method.

To check the efficacy of the PCA method in larger samples, I also simulated example V. from Feizi et al. (2013). This example contains 5 time series, so it is meaningful to calculate the principal components.

The structure of the procedure was the same, as in the former case. I choose different sample sizes ($T = 100, 250, 500, 1000, 3000, 5000$), and I run the Monte Carlo simulation with 1000 repetitions in the first case and 500 repetitions in the second case.

	Unconditional test from leave-one-out approach		PCA approach of Song and Taamouti (2019)	
T	$a_{2,1}^{(4)}$	$a_{3,2}^{(4)}$	$a_{2,1}^{(4)}$	$a_{3,2}^{(4)}$
100	0.274	0.981	0.028	0.006
250	0.746	1	0.418	1
500	0.984	1	0.914	1
1000	1	1	1	1
3000	1	1	1	1
5000	1	1	1	1

Notes: Coefficient $a_{u,v}^{(K)}$ means that institution v Granger-causes institution u. K only indicates the number of example, while $a_{u,v}^{(K)}$ measures the coefficient. But in this exercise I only highlighted the existance of the given relationship, not its strength.

Table 32: Hit ratio of different methods in the modified example of Song and Taamouti (2019)

	Unconditional test from leave-one-out approach						PCA approach of Song and Taamouti (2019)					
T	$a_{2,1}^{(5)}$	$a_{3,2}^{(5)}$	$a_{4,2}^{(5)}$	$a_{4,3}^{(5)}$	$a_{5,3}^{(5)}$	$a_{5,4}^{(5)}$	$a_{2,1}^{(5)}$	$a_{3,2}^{(5)}$	$a_{4,2}^{(5)}$	$a_{4,3}^{(5)}$	$a_{5,3}^{(5)}$	$a_{5,4}^{(5)}$
100	0.355	0.443	0.436	0.446	0.488	0.555	0.01	0.009	0.008	0.01	0.009	0.014
250	0.601	0.668	0.69	0.707	0.725	0.791	0.237	0.221	0.237	0.265	0.313	0.312
500	0.771	0.795	0.811	0.839	0.84	0.892	0.811	0.844	0.877	0.875	0.893	0.91
1000	0.851	0.858	0.854	0.903	0.902	0.94	1	1	1	1	1	1
3000	0.908	0.93	0.924	0.949	0.937	0.961	1	1	1	1	1	1
5000	0.941	0.953	0.947	0.96	0.95	0.977	1	1	1	1	1	1

Notes: Coefficient $a_{u,v}^{(K)}$ means that institution v Granger-causes institution u. K only indicates the number of example, while $a_{u,v}^{(K)}$ measures the coefficient. But in this exercise I only highlighted the existance of the given relationship, not its strength.

Table 33: Hit ratio of different methods in the example of Feizi et al. (2013)

The output table 33 shows similar tendencies, then in a smaller one. In small sample sizes ($T = 100, 250$), the framework of Song and Taamouti (2019) underperforms the unconditional version of the leave-one-out approach. However, the hit ratios are a little bit better in the medium sample size ($T = 500$). Utilizing larger samples demonstrates the advantages of the PCA based methodology, which makes it suitable for large sample analysis. Only one thing must be considered. The widely used 250 day-long window cannot be applied with enough confidence. The simulation results are in accordance with the widely spread exercise: the number of observations is recommended to exceed the quintuple of the variables.

C.3 ΔCoVaR

The literature summary pointed out that the generally used methodologies are ΔCoVaR, MES and SRISK. For producing robust results, it is inevitable to contrast the econometric solution to the widely spread frameworks.

The ΔCoVaR framework is not only comparable with the causality approach, while it is a comprehensive methodology, but like Berdin and Sottocornola (2015) observed similar results using both methods.

The Conditional Value-at-risk (CoVaR) was proposed by Adrian and Brunnermeier (2011). The CoVaR framework is based on the definition of Value-at-Risk.

$$Pr(R^i \leq VaR_q^i) = q \tag{66}$$

VaR_q^i assigns that institution i will suffer a given level of loss or higher for a target horizon - generally one year - with q% probability.

Consider two institutions, institution i and institution j. $\mathbb{C}(R^i)$ summarizes the event set affecting institution i, which usually means the VaR_q^i. So, $CoVaR_q^{i|j}$ represents the q^{th} quantile of the conditional probability distribution of institution j.

$$Pr(R^j \leq CoVaR_q^{j|\mathbb{C}(R^i)}) = q \tag{67}$$

A formal property of the CoVaR is that in general $CoVaR_q^{i|j} \neq CoVaR_q^{j|i}$, which is similarity with the Granger-causality.

Economically, CoVaR means that if institution i suffers a loss generated by event (R^i), then institution j will suffer higher or equal loss with q% probability thanks to institution i. So, CoVaR can express pairwise connections among financial institutions. However, the approach misses the reference point, which helps to judge the seriousness of a shock. ΔCoVaR corrects this deficiency of CoVaR comparing the level of the shock to the median ("normal") state of the institution, formally:

$$\Delta CoVaR_q^{j|i} = CoVaR_q^{j|X^i=VaR_q^i} - CoVaR_q^{j|X^i=Median^i} \tag{68}$$

Typically, the index j denotes the financial system, and the ΔCoVaR measures the marginal contribution of institution i to the systemic risk. Adrian and Brunnermeier (2011) proposed ΔCoVaR to measure the spillovers between institutions, while these effects are responsible for risk amplification. Also, the measure is specified on risk transmission quantification, and selecting the j entity properly, will make it applicable for the analysis of the different levels of the market (individual institutions, sectors and whole industries). Furthermore, the Conditional Value-at-Risk method is suitable for cross-sectional and time-varying data sets. You can see the detailed description of the time-varying ΔCoVaR in the Appendix C.4.

C.4 Estimation of ΔCoVaR

Adrian and Brunnermeier (2011) proposed the Conditional Value-at-Risk (CoVaR) framework, which is adequate for both cross-sectional and time-varying datasets. I will propose only the estimation of the time-varying version, followed by Adrian and Brunnermeier (2011). Thus the cross-sectional modification assumes that CoVaR is constant for a given time horizon, which is not an adequate assumption in distressed periods when market conditions change suddenly. So, it is not surprising that financial econometricians prefer the time-varying framework.

The steps of the estimations are the followings (Adrian and Brunnermeier 2011, Bernal et al. 2014, p.14-15 and p.273-275):

1. Use quantile regression to estimate tail behaviour of i^{th} time series (69).

$$R_t^i(q) = \alpha_q^i + \gamma_q^i M_{t-1} + \epsilon_t^i \tag{69}$$

   α_q^i and γ_q^i are constants, M_{t-1} represents the lagged value of the state variables - specified later -, and ϵ_t^i error term is assumed to be i.i.d. probability variable with zero mean and unit variance.

2. Similarly, compute the q^{th} VaR for institution i at time t.

$$\widehat{VaR_t^i}(q) = \hat{\alpha}_q^i + \hat{\gamma}_q^i M_{t-1}, \tag{70}$$

   where $\hat{\alpha}_q^i$ and $\hat{\gamma}_q^i$ are the estimated parameters in the equation (69)

3. Model the behaviour of th q^{th} quantile of the system variable at time t:

$$R_t^{system}(q) = \alpha_q^{system|i} + \beta_q^{system|i} \widehat{VaR}_t^i + \gamma_q^{system|i} M_{t-1}. \tag{71}$$

   The state of the system is modelled individually for all time series (R_t^i). Also state variable are included (M_{t-1}), and $\alpha_q^{system|i}, \beta_q^{system|i}, \gamma_q^{system|i}$ constants and $\epsilon_t^{i|system}$ innovation.

4. Similarly, use the estimated parameters from equation (71) to compute the CoVaR of the system:

$$\widehat{CoVaR}_t^{system}(q) = \alpha_q^{system|i} + \beta_q^{system|i} R_t^i + \gamma_q^{system|i} M_{t-1} + \epsilon_t^{i|system}. \tag{72}$$

5. Next step is the quantification of $\Delta\widehat{CoVaR}$, you need both the median and the selected q^{th} quantile of the system CoVaR.

$$\begin{aligned} \Delta\widehat{CoVaR}_t^{system|i}(q) =& \widehat{CoVaR}_t^{system|i}(q) - \widehat{CoVaR}_t^{system|i}(50\%) = \\ & \beta_q^{system|i}(\widehat{VaR}_t^{system|i}(q) - \widehat{VaR}_t^{system|i}(50\%)) \end{aligned} \tag{73}$$

As a remark I should add that ΔCoVaR is calculated on company returns time series, as far as I know, it was never used on volatility data. Furthermore, ΔCoVaR values are, in general, negative while it expresses the highest losses. Naturally, the higher the possible loss is, the more important the individual institution is, thus it prose higher systemic risk.

Adrian and Brunnermeier (2011) suggested a few state variables (Adrian and Brunnermeier 2011, p.15-16):

- VIX index, which captures the implied volatility
- Short term liquidity spread, measured by the difference of the three-month repo rate and the three-month bill rate.
- The change in the three-month Treasury bill rate, which explains the tails of financial sector market-valued asset returns.
- The change in the slope of the yield curve, defined by the yield spread of the ten-year Treasury rate and the three-month bill rate.
- The change in the credit spread between BAA-rated bonds and the Treasury rate.
- Weekly equity market return from CRSP.
- Weekly real estate sector return above the market return.[16]

[16]The last to items should be adapted to the analysed markets and data types, like volatility.

C.5 MES

Marginal Expected Shortfall (MES) summarizes at the institutional level the average return for the 5% worst days of the market, which makes this measure simply to calculate and interpret.

$$MES^i_{5\%} = \frac{1}{\#days} \sum_{\text{t: system in 5\% tail}} R^i_t, \tag{74}$$

where R_t are the returns of institution i.

The marginal expected shortfall was introduced by Acharya et al. (2010) based on systemic expected shortfall (SES), which was decomposed to leverage and marginal expected shortfall (Acharya et al. 2010, 2012).

To better understand the marginal expected shortfall, it useful to derive its formula from the expected shortfall (ES).

Let consider the equation (75) characterizing the expected shortfall, which quantifies the expected loss if the loss exceeds $-VaR_q$ in the lower tail.

$$ES^i_q = -\mathbf{E}(R^i | R^i \leq -VaR^i_q) \tag{75}$$

Assume that institution i has a portfolio with returns of r_j and portfolio weights of y_j, then the return of institution is $R^i = \sum_j y_j r_j$. So the expected shortfall can formulate as follows:

$$ES^i_q = -\sum_j y_j \mathbf{E}(r_j | R^i \leq -VaR^i_q). \tag{76}$$

Then the risk exposure of institution i to institution j can be seen in equation (77).

$$\frac{\partial ES^i_q}{\partial y_j} = \mathbf{E}(r_j | R^i \leq -VaR^i_q) \equiv MES^{i|j}_q \tag{77}$$

$MES^{i|j}$ measures the risk contribution of institution i to the overall exposure of institution j, where j is selected in general the whole market. (In that case, MES^i_q symbolises the contribution of institution i to the systemic risk of the whole market. More insights about the empirical calculation of the MES you can see in the Appendix C.6.))

C.6 Estimation of MES

The dynamic version of marginal expected shortfall is widely used in order to quantify risk exposure of institutions based on Acharya et al. (2010, 2012), Brownlees and Engle (2017).

Consider R_t^j and R_t^M as the time series of institution j and the market (e.g. returns, volatilities, CDS etc.)[17]

$$\begin{aligned} R_t^M &= \sigma_t^M \epsilon_{t,1}^M \\ R_t^M &= \sigma_t^j \rho_t^j \epsilon_{t,2}^M + \sigma_t^M \sqrt{1-(\rho_t^j)^2}\epsilon_{t,2}^j \\ \left(\epsilon_{t,1}^M, \epsilon_{t,2}^j\right) &\sim H, \end{aligned} \tag{78}$$

where σ_t^j and σ_t^M are the conditional volatility of the institution j and the market, while ρ_t^j is the conditional correlation between institution j and the market.

The bivariate process of the error term $\left(\epsilon_{t,1}^M, \epsilon_{t,2}^j\right)$ is a i.i.d. vector variable with zero mean, unit variance and zero covariance ($\mathbb{E}(\epsilon_{t,k}^i) = 0$, $Var(\epsilon_{t,k}^i) = 1$, i$\in j, M$ and k $\in 1, 2$). So, thanks to the joint distribution of the innovation (H), the error terms are uncorrelated, but in general not independent.

The one period-ahead MES can be rewritten as follows:

$$\begin{aligned} MES_{q,t-1}^j(1) = \mathbb{E}_{t-1}(R_t^j | R_t^M \leq -VaR_q^M) &= \sigma_t^j \mathbb{E}_{t-1}\left(\rho_t^j \epsilon_{t,1}^M + \sqrt{1-(\rho_t^j)^2}\epsilon_{t,2}^j \middle| \frac{-VaR_q^M}{\sigma_t^M}\right) \\ &= \sigma_t^j \rho_t^j \mathbb{E}_{t-1}\left(\epsilon_{t,1}^M \middle| \frac{-VaR_q^M}{\sigma_t^M}\right) + \sigma_t^j \sqrt{1-(\rho_t^j)^2}\mathbb{E}_{t-1}\left(\epsilon_{t,2}^j \middle| \frac{-VaR_q^M}{\sigma_t^M}\right), \end{aligned} \tag{79}$$

where σ_t^M and σ_t^i are estimated by a GJR-GARCH modell, while ρ_t^j is characterized by a dynamical conditional correlation modell.

C.7 SRISK

Brownlees and Engle (2017) proposed SRISK to gauge the capital shortfall institutions are facing if a serious systemic event happens. To introduce SRISK, let define firstly the capital shortfall (CS) for institution i at time t:

$$CS_t^i = k(D_t^i + W_t^i - W_t^i), \tag{80}$$

where W_t^i gauges the market capitalisation of institution i at time t, D_t^i the current liabilities,

[17] I follow the formalism of Weiß and Mühlnickel (2014) (Weiß and Mühlnickel 2014, p.112)

while k expresses the prudential capital ratio fixed at 8% level. Capital shortfall takes positive values in distressed periods, while negative during normal times.

Brownlees and Engle (2017) constructed SRISK based on the capital shortfall conditional on a systemic event C for the time horizon h.[18] Formally equation (81) assigns the correct mathematical formula for SRISK of institution i at time t.

$$SRISK_t^i = \mathbb{E}_t(CS_{t+h}^i | R_{t+1:t+h}^M < C) = k\mathbb{E}_t(D_{t+h}^i | R_{t+1:t+h}^M < C) - (1-k)\mathbb{E}_t(W_{t+h}^i | R_{t+1:t+h}^M < C), \tag{81}$$

where $R_{t+1:t+h}^M$ denotes the arithmetic mean of the market portfolio between $t+1$ and $t+h$. Equation (81) demonstrates the theoretical form of SRISK, but for empirical analysis is handier to look equation (82).

$$SRISK_t^i = \mathbb{E}\Big(\Big(k(D_t^i + W_t^i) - W_t^i\Big)\Big| Crisis\Big) = kD_t^i - (1-k)(1 - LRMES_{q,t}^i)W_t^i, \tag{82}$$

while $LRMES_t^i$ the long run marginal expected shortfall for company i at time t. LRMES quantifies the expected loss of the equity in stress scenarios. Which can be approximated as $1 - \exp(-18MES_{q,t}^i)$.

Economically, SRISK depends on the firm size and the leverage, which are essential contributing factors to systemic risk, which usually captures company-related characteristics and accordingly can provide new information about the companies.

SRISK can also express the contribution of individual companies at time t to systemic risk as (Acharya et al. 2012, p.61):

$$SRISK_t^i\% = \frac{SRISK_t^i}{\sum_{j:SRISK_t^j>0} SRISK_t^j} \tag{83}$$

Equation (83) is convenient for creating rankings from the systemically important institutions representing a clear order of risk spillovers.

[18] The C threshold was set to 10%, h to 22 days (Brownlees and Engle 2017, p.52)