Pricing Portfolios Of Financial Products

Vanilla Options

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Overview

1. Introduction
2. Binomial Pricing Model
3. Black-Scholes Model
4. LSM Method

Citi Project

Goal Of The Project

Pricing investment portfolios of financial products as efficiently as possible based on a large number of future scenarios.

This Project is expected to last for three semesters.

During this semester, I was exposed to the financial background needed for this project, along with multiple option pricing models that will be discussed in this presentation.

Classes of Financial Instruments

- The underlying assets: shares, bonds, foreign currencies, and commodities.
- 'Derivatives' tied to them -contracts that guarantee some payment or delivery in the future contingent on the behavior of the underlying stock-

Definition

Option: A contract that gives to its owner, the right, but not the obligation, to buy or sell a specific quantity of an underlying asset at a specified strike price

Financial Derivatives

Types Of Options

- **Call** options: they give their holder the right to *buy* the underlying asset by a certain date for a certain price. The payoff of a Call option is given by

$$(S(T) - K)^+$$

  where T refers to the expiry date of the option and K is the strike price of the option.
- **Put** options: they give their holder the right to *sell* the underlying asset by a certain date for a certain price. The payoff of a Put option is given by $(K - S(T))^+$

American vs European Options

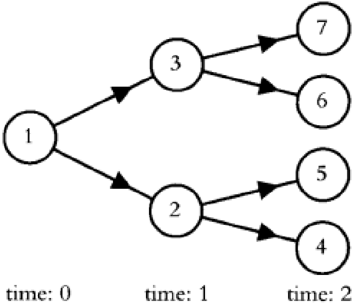
American options can be exercised at *any time* up to the expiration date, while **European** options can be exercised only on the expiration date itself.

Mathematical Framework

The financial market of interest is described by complete probability space $(\Omega, \mathcal{F}, P)$ and finite time horizon $[0, T]$.

- The state space Ω is the set of all possible realizations of the economy between times 0 and T
- typical element $\omega \in \Omega$ represents a sample path
- $\mathcal{F}$ is the sigma algebra of events at time T
- P is a probability measure defined on the elements of $\mathcal{F}$

Binomial options pricing model



The underlying assumption is that the stock price follows a random walk. In each time step δt, it has a certain probability p_u of moving up by a certain percentage amount u and a certain probability $p_d = 1 - p_u$ of moving down by a certain percentage d.

As the time step $\delta t \to 0$ becomes smaller, this model approaches the Black–Scholes–Merton model.

Value of an option in a 1-step model

Value of the option

$$V = exp(-r\delta t)\left((1-q)\,f(d) + qf(u)\right) \qquad \text{where} \quad q = \frac{exp(r\delta t) - d}{u - d}$$

This is the discounted expected value of the claim under the risk-neutral measure q.

Generalization

After calculating the value of the option at the final layer of the tree, we traverse backwards through the layers of the tree, calculating the value of the claim at each node similar to what we did in the one-step model, until we reach the root of the tree.

Value of an American option

The value of the option is (maximised over all possible stopping strategies):

$$sup_\tau \mathbb{E}_{\mathbb{Q}} \left(e^{-r\tau} (S_\tau - K)^+ \right)$$

where τ is a stopping time, so the problem is determining the optimal stopping time.

The valuation procedure is to work back through the tree from the end to the beginning, testing at each node to see whether early exercise is optimal. At earlier nodes the value of the option is the greater of the payoff from early exercise and the discounted expected value (under the risk free rate) of future payoffs.

Black-Scholes Model

In this model, the stock is modelled by a geometric Brownian motion.

Model

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right).$$

There is an explicit formula to calculate the price of European put and call options.

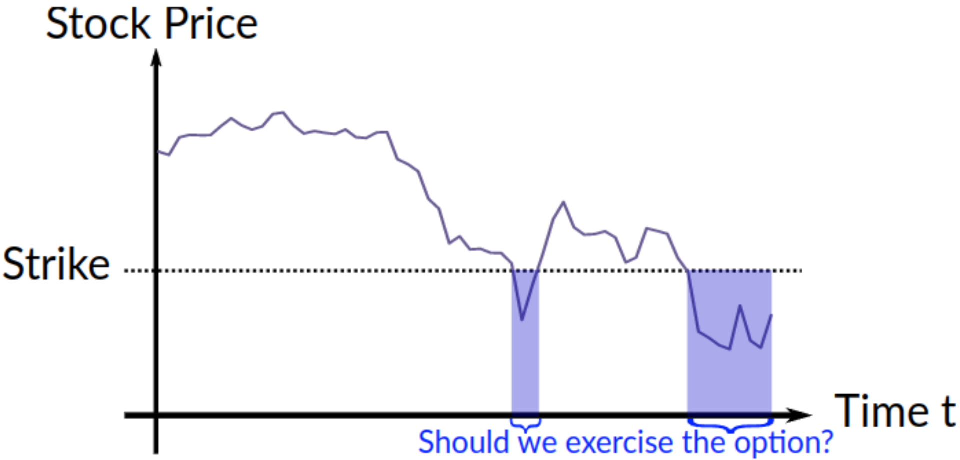
Longstaff Schwartz

LSM uses least-squares regression on a finite set of functions to approximate conditional expectation values.

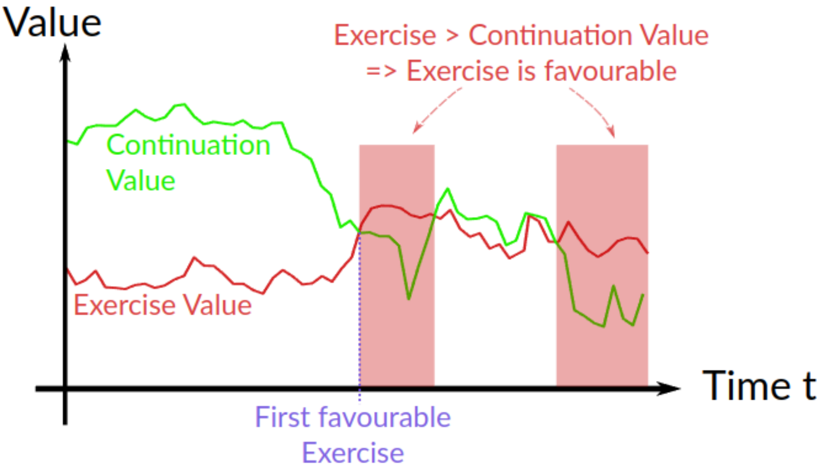
- We only allow early exercise at discrete times $0 < t_1 < t_2 < \cdots < t_N = T$.
- For a given time t_j, decision to perform early is exercise is carried out if the payoff from immediate exercise exceeds the continuation value

We start the algorithm by generating multiple paths representing the behaviour of the stock over the time horizon.

Continuation



Continuation



LSM Algorithm

1. Determine the cashflow vector C_{t_N} at the last time step T_N
$$C_{i,t_N} = (S_{i,t_N} - K)^+$$
2. We consider the spot prices at time-step t_{N-1}, selecting only the in-the-money paths, i.e:
$$\left(S_{i,t_{N-1}} - K\right)^+ > 0$$
3. Regress the discounted future cash-flows onto a finite set of basis functions to obtain the continuation values.
$$Cont_{i,t_{n-1}} = \sum_j a_j(t_n)B_j(S)$$
4. Once both continuation and exercise values are ready, early exercise is performed if:
$$C_{i,t_{n-1}} > Cont_{i,t_{n-1}}$$
5. Once we finish our backward iteration process and reach the initial point, we can build a cashflow or value matrix, and the option value is given by the arithmetic average of the values.

References

- Baxter, Martin, and Andrew Rennie. Financial Calculus: An Introduction to Derivative Pricing. Cambridge University Press, 2014.
- Hull, John. Options, Futures, and Other Derivatives. 2017.
- Longstaff, Francis Schwartz, Eduardo. (2001). Valuing American Options by Simulation: A Simple Least-Squares Approach. Review of Financial Studies. 14. 113-47. 10.1093/rfs/14.1.113.

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