1. A reminder: a ring R is *(left) hereditary* if and only if every homomorphic image of every injective (left) module is injective (for example, $\mathbb{Z}$ is hereditary). (Note that it can be shown that this is equivalent to the fact that submodules of projective modules are projective.) Prove that if R is hereditary then for every $M, N \in R$-Mod left R-module $\mathrm{Ext}^k_R(M,N) = 0$ for all $k > 1$.

2. a) Let A, B be cyclic abelian groups. Determine the groups $\mathrm{Ext}^1_{\mathbb{Z}}(A,B)$.
b) Do the same when A and B are arbitrary finite abelian groups.

3. Let $0 \to K_n \to P_{n-1} \to \cdots \to P_1 \to P_0 \to M \to 0$ be exact and suppose that P_i is projective for all i. Prove that $\mathrm{Ext}^k_R(M,N) \simeq \mathrm{Ext}^{k-n}_R(K_n, N)$ for every $k > n$.

4. Suppose that the resolution in the previous problem is minimal and N is simple. Prove that $\mathrm{Ext}^n_R(M,N) \simeq \mathrm{Hom}_R(K_n, N)$.

5. Let $A_A = \begin{smallmatrix} & 1 & \\ & 2 & \\ 1 & & 3 \end{smallmatrix} \oplus \begin{smallmatrix} & 2 & \\ 1 & & 3 \\ & & 4 \end{smallmatrix} \oplus \begin{smallmatrix} 3 \\ 4 \end{smallmatrix} \oplus \begin{smallmatrix} 4 \\ 1 \end{smallmatrix}$.
a) Draw a graph Γ and admissible ideal I of relations, so that $A \simeq K\Gamma/I$.
b) Determine the dimension of $\mathrm{Ext}^3_A(\,1\,,\,1\,)$.

6. Let $A_A = \begin{smallmatrix} & 1 & \\ & 2 & \\ 1 & & 3 \end{smallmatrix} \oplus \begin{smallmatrix} & 2 & \\ 1 & & 3 \\ & & 4 \end{smallmatrix} \oplus \begin{smallmatrix} 3 \\ 4 \\ 1 \end{smallmatrix} \oplus \begin{smallmatrix} 4 \\ 1 \end{smallmatrix}$.
a) Describe A as a path algebra modulo some relations.
b) Compute $\dim \mathrm{Ext}^3_A(\,1\,,\begin{smallmatrix} 2 \\ 1 \end{smallmatrix})$.
c) Show that for every $n \in \mathbb{N}$ there exists $k > n$, such that $\dim \mathrm{Ext}^k_A(\,1\,,\begin{smallmatrix} 2 \\ 1 \end{smallmatrix}) \neq 0$.

7. Give a new proof for the fact that every projective module is flat.

8. Give a short exact sequence of Abelian groups $0 \to \mathbb{Z}_4 \to M \to \mathbb{Z}_4 \to 0$ which is not split, but the middle term M is not indecomposable.

9. Let us take the graph $\Gamma:\quad 1 \to 2 \leftarrow 3 \to 4$ and the path algebra $A = K\Gamma$.
a) Find an indecomposable (right) A module M of composition length 4 (there is only one such module).
b) Let N be the simple module corresponding to vertex 3. Show that $\dim(\mathrm{Ext}^1_A(N,M)) = 1$, and describe the middle term of the nonzero elements in this extension space.
c) Find as many indecomposable A-modules as you can.

10. Give a characterization of the fact that a short exact sequence is split in terms of homomorphisms, hence conclude that any additive functor preserves split exact sequences.