1. (Hereditary rings revisited:) Prove that the following statements are equivalent for a ring R:

(i) every submodule of every projective left R-module is projective;

(ii) every left ideal of R is projective;

(iii) every homomorphic image of every injective left R-module is injective (i. e. R is left hereditary);

(iv) the left global dimension of R is at most 1.

2. a) Let Γ be a graph for which the path algebra $K\Gamma$ is finite dimensional. Prove that $K\Gamma$ is (left and right) hereditary.

b*) Prove the same statement without the assumption on the dimension of $K\Gamma$.

3. Consider the Ext^3-spaces of problems #2/6 and #2/7. Represent a non-zero element of these spaces by an exact sequence of length 3.

4. Decide whether the following exact sequence in $\mathrm{Ex}^2_A(\,{\scriptstyle 1}\,,\,{\scriptstyle 3}\,)$ is equivalent to the 0 element

$$0 \to {\scriptstyle 3} \to {\scriptstyle \begin{smallmatrix}2\\3\end{smallmatrix}} \to {\scriptstyle \begin{smallmatrix}1\\2\end{smallmatrix}} \to {\scriptstyle 1} \to 0$$

when the regular representation of the algebra can be described as follows:

(i) ${}_AA = {\scriptstyle \begin{smallmatrix}1\\2\\3\end{smallmatrix}} \oplus {\scriptstyle \begin{smallmatrix}2\\3\end{smallmatrix}} \oplus {\scriptstyle 3}$; (ii) ${}_AA = {\scriptstyle \begin{smallmatrix}1\\2\end{smallmatrix}} \oplus {\scriptstyle \begin{smallmatrix}2\\3\end{smallmatrix}} \oplus {\scriptstyle 3}$.

5. Let A be a finite dimensional (left) hereditary algebra and A^* its Yoneda-extension algebra: this means that if $\hat{S}$ is a semisimple module which is the direct sum of all isomorphism types of simple modules over A, then $A^* = \bigoplus_{i\geq 0} \mathrm{Ext}^i_A(\hat{S}, \hat{S})$ as a vector space and the multiplication is defined via the Yoneda product. Show that in this case $J(A^*)^2 = 0$.

6*.** Suppose A is an abelian group for which $\mathrm{Ext}^1_{\mathbb{Z}}(A, \mathbb{Z}) = 0$. Is it true that A is necessarily free?

7. Prove that if A is a torsion abelian group, then $\mathrm{Ext}^1_{\mathbb{Z}}(A, \mathbb{Z}) \simeq \mathrm{Hom}_{\mathbb{Z}}(A, \mathbb{R}/\mathbb{Z})$.

8. Let A be a finite dimensional K-algebra for which ${}_AA$ is injective. (Such an algebra is also called a *quasi-Frobenius algebra.*) Prove that if for a module $M \in A$-Mod we have $pd\, M < \infty$ then $pd\, M = 0$ i. e. M is projective.

9. Take the graph $1 \overset{\alpha}{\underset{\beta}{\rightrightarrows}} 2$ with $2 \xrightarrow{\gamma} 1$ and take the path algebra modulo relations $K\Gamma/I$ where $I = (\alpha\gamma, \gamma\beta)$. Compute the (left) global dimension of A.

10. Prove that for an arbitrary ring R we have $l\ gl\ dim\, R = l\ gl\ dim\, M_n(R)$.