

1. Let $A = K\Gamma/I$ for some finite graph Γ with $\Gamma = (V, E)$ and $V = \{1, 2, \dots, n\}$. As usual, let $S(i)$ be the i 'th simple right A -module. Show that the number of arrows $i \rightarrow j$ in Γ is equal to $\dim_K \text{Ext}_A^1(S(i), S(j))$.
2. Find the Gabriel quiver of the following group algebras:
 - a) $\mathbb{F}_2[V]$, where V is the Klein group;
 - b) $\mathbb{F}_3[S_3]$, where S_3 is the symmetric group on 3 letters.Determine, whether the corresponding group algebra is a path algebra modulo some relations.
3. Show that:
 - a) $\mathbb{Z}_{p^n} \hookrightarrow \mathbb{Z}_{p^{n+1}} \oplus \mathbb{Z}_{p^{n-1}}$ is left minimal left almost split and
 - b) $\mathbb{Z}_{p^{n+1}} \oplus \mathbb{Z}_{p^{n-1}} \twoheadrightarrow \mathbb{Z}_{p^n}$ is right minimal right almost split in the category of Abelian groups.
4. Check which of the following algebras are Nakayama algebras:
 - a) $K[x]/(x^n)$, where $n \geq 2$;
 - b) $T_n(K)$, the ring of upper triangular matrices over K ;
 - c) $K[\mathbb{Z}_{p^n}]$, where K is a field of characteristic p ;
 - d) $\mathbb{F}_2[V]$, where V is the Klein group.