Finding the maximum and minimum elements with lies

Dömötör Pálvölgyi*

Eötvös University (ELTE), Budapest

Katona '70

*Mostly joint work with Dániel Gerbner, Balázs Patkós and Gábor Wiener

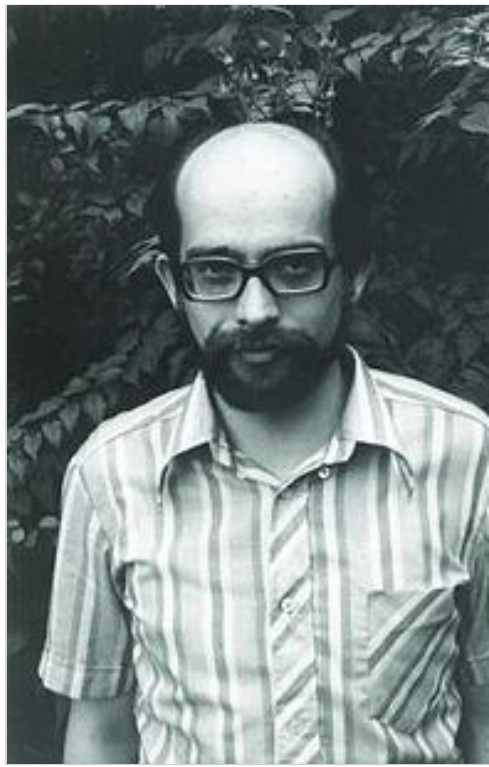
Gyula O. H. Katona – Wikipedia tiếng Việt http://vi.wikipedia.org/wiki/Gyula_

Gyula O. H. Katona

Bách khoa toàn thư mở Wikipedia

Gyula O. H. Katona sinh ngày 16.3.1941 là nhà toán học người Hungary, nổi tiếng về công trình nghiên cứu lý thuyết tổ hợp và nhất là về Định lý Kruskal–Katona cùng việc chứng minh định lý Erdős–Ko–Rado của ông.

Katona là tổng thư ký của Hội Toán học János Bolyai từ năm 1990 tới 1996. Năm 1966 và 1968 ông đoạt Giải Grünwald của Hội Toán học János Bolyai dành cho các nhà toán học trẻ xuất sắc. Năm 1976, ông đoạt Giải Alfréd Rényi của Viện Toán học Alfréd Rényi. Năm 2006, ông được bầu vào Hàn lâm viện Khoa học Hungary.



Gyula Katona tại Erlangen, 1975.

Gia đình

Gyula O.H. Katona là cha của Gyula Y. Katona, cũng là nhà toán học chuyên về lý thuyết tổ hợp như người cha.

70 (số)

Bách khoa toàn thư mở Wikipedia

Bảy mươi (70) là một số tự nhiên ngay sau 69 và ngay trước 71.

Lấy từ "http://vi.wikipedia.org/wiki/70_(s%E1%BB%91)"
Thể loại: Số nguyên

<< **70** 71 72 73 74 75 76 77 78 79 >>

0 10 20 30 40 50 60 **70** 80 90 >>

70	
Phân tích nhân tử	2 · 5
Số La Mã	LXX
Mã Unicode của số La Mã	
Biểu diễn theo Hệ nhị phân	1000
Biểu diễn theo Hệ thập lục phân	46

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Katona 170?

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Katona 1970?

Gyula, watch out, your students are rising!

*Mostly joint work with Dániel Gerbner, Balázs Patkós and Gábor Wiener
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Beamer is messing with you.

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You have no joint paper?

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You have not taken any of his lectures?

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Maybe you should start the talk...

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In our model - Fixed number of lies, best worst-case algorithm, questions can depend on previous answers.

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We are given an ordered set of size n (with no equal elements) and we can use pairwise comparisons.

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Finding the biggest and the smallest with k lies - Our main topic.

Finding the biggest and the smallest with k lies

Theorem (Aigner)

$\boxed{\textit{General } k} \leq (k + \Theta(\sqrt{k}))n + \Theta_k(1).$

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Does "10" $\to 0$ as $k \to \infty$?

Theorem (P.)

No. $\boxed{\textit{General } k} \geq (k + 1.5)n + \Theta_k(1).$

Terminology

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teams - elements of the set

Terminology

teams - elements of the set

match - comparison

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win - bigger in comparison

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comparison graph - digraph, ab is an edge iff a has beaten b

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Rule of thumb - The evil Adversary never lies.

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Rule of thumb - The evil Adversary never lies.
Except maybe at the end.

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Claim

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Rule of thumb - No lies until the end.

Lower bound: $< (k+1)(n-1)+k$ questions are not enough:

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Proof.

Reply without lies until there are only two teams with in-degree $\leq k$ and one of them has in-degree 0, the other k.

After this we "change our mind" and reply such that the team with in-degree k always wins. □

General way to increase lower bound by k for k lies

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Consistency $\Rightarrow$ All answers are true for some x.

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Answer next k questions according to y. $\square$

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Game ends when there is only one pebble of each color.

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Adversary can answer such that only matches between two newbies result in loss of two pebbles $\Rightarrow \lceil \frac{3n}{2} \rceil - 2$ questions are needed. □

Natural algorithm for 1 lie

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Give every team 2 red and 2 blue pebbles.

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Match teams with the same set of pebbles until possible.

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After this at most 8 teams remain with at least one pebble, enough to compare them.

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After this at most 8 teams remain with at least one pebble, enough to compare them.

How many comparisons are needed in total?

Potential function p

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$p(a, b)$ measures number of questions needed in average for a team with a red and b blue pebbles to lose all its pebbles.

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Not optimal!

CONGRATULATIONS, YOU HAVE MADE IT HALF WAY THROUGH THE TALK!

For myself: I should check how much time is left.

CONGRATULATIONS, YOU HAVE MADE IT THROUGH ONE MORE SLIDE!

For myself: Check time if not done before.

CONGRATULATIONS, YOU HAVE MADE IT THROUGH YET ANOTHER SLIDE!

For myself: Stop this and go on with your talk.

BUT WHY NOW, WE MADE IT THROUGH ANOTHER SLIDE!

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PEOPLE PREFER THIS TO YOUR PATHETIC TALK...

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AS YOU WISH...

Main idea of improved algorithm for 1 lie

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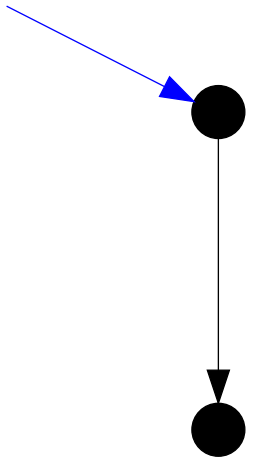

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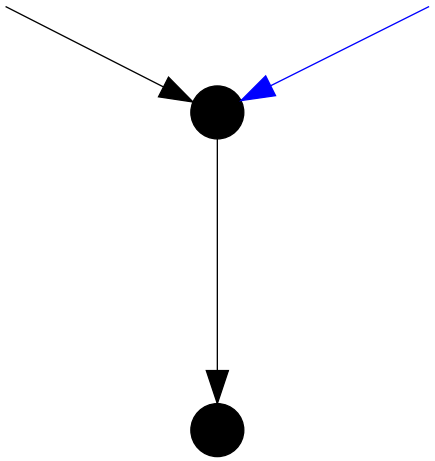

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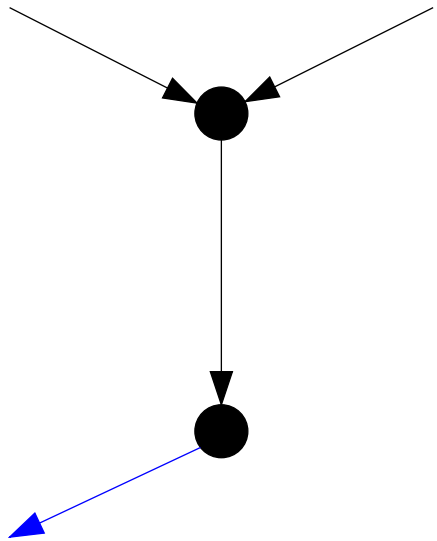

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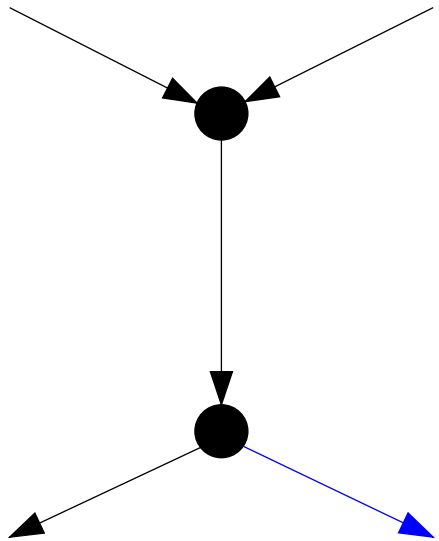

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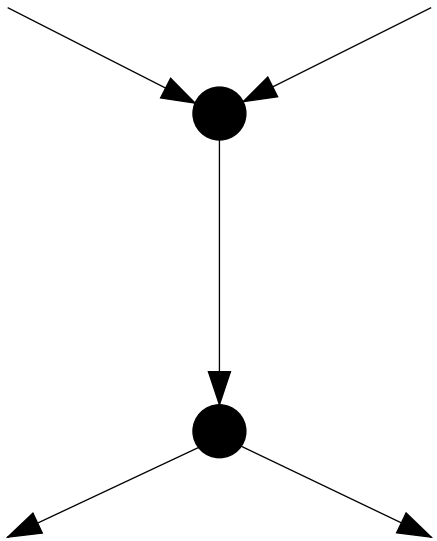

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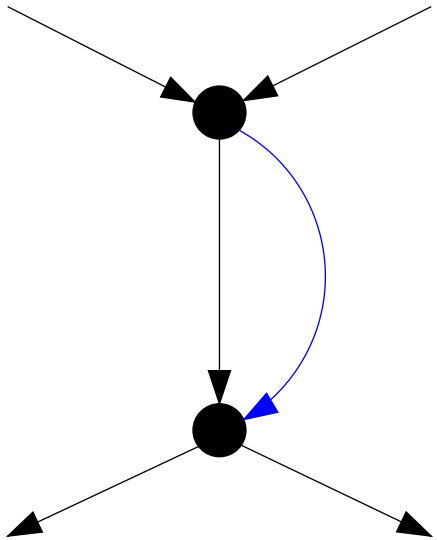

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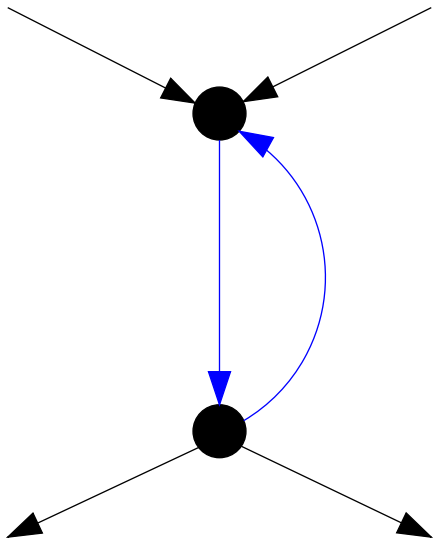

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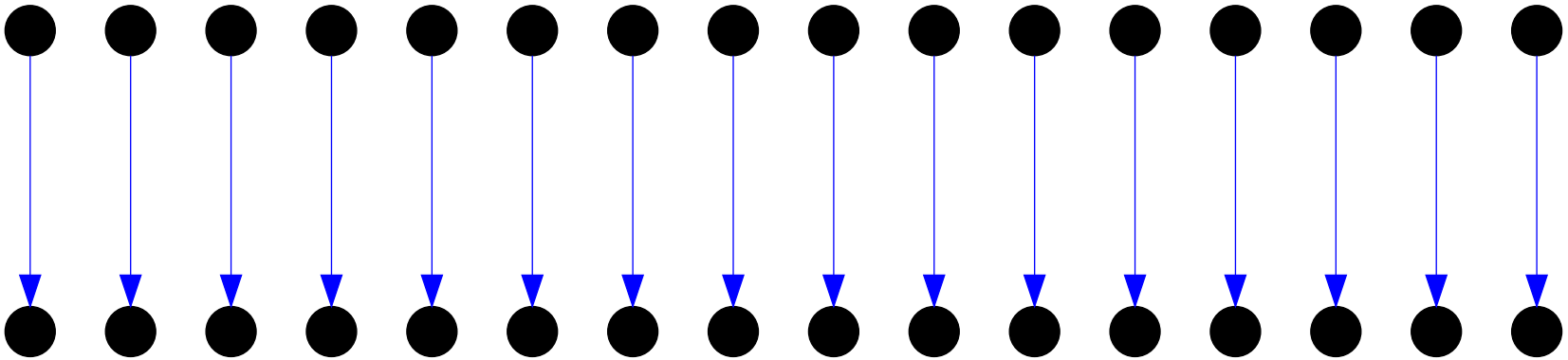

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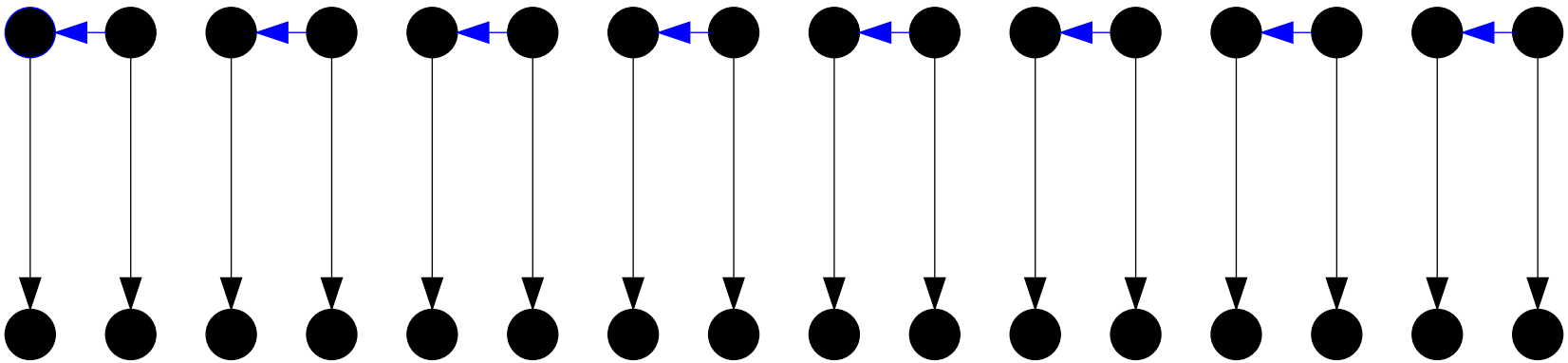

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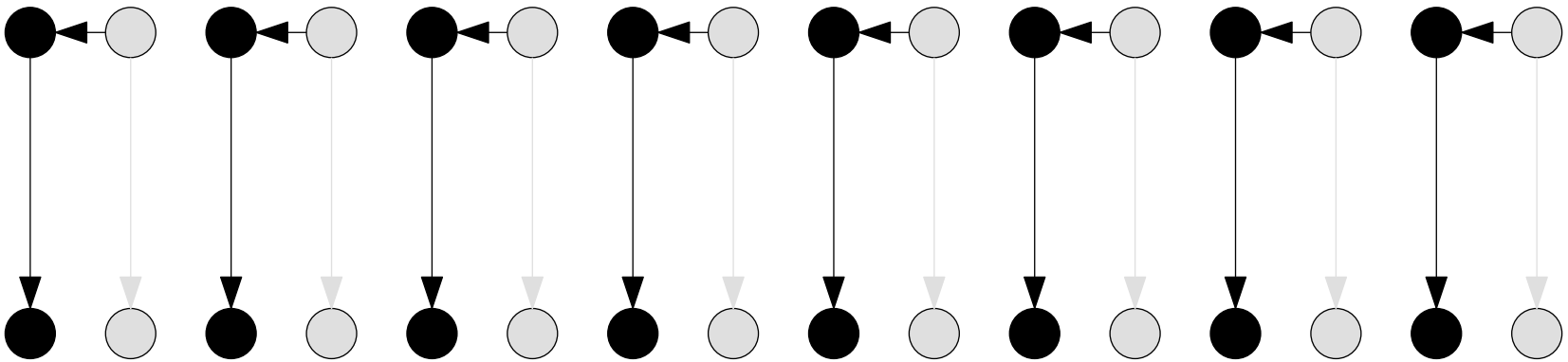

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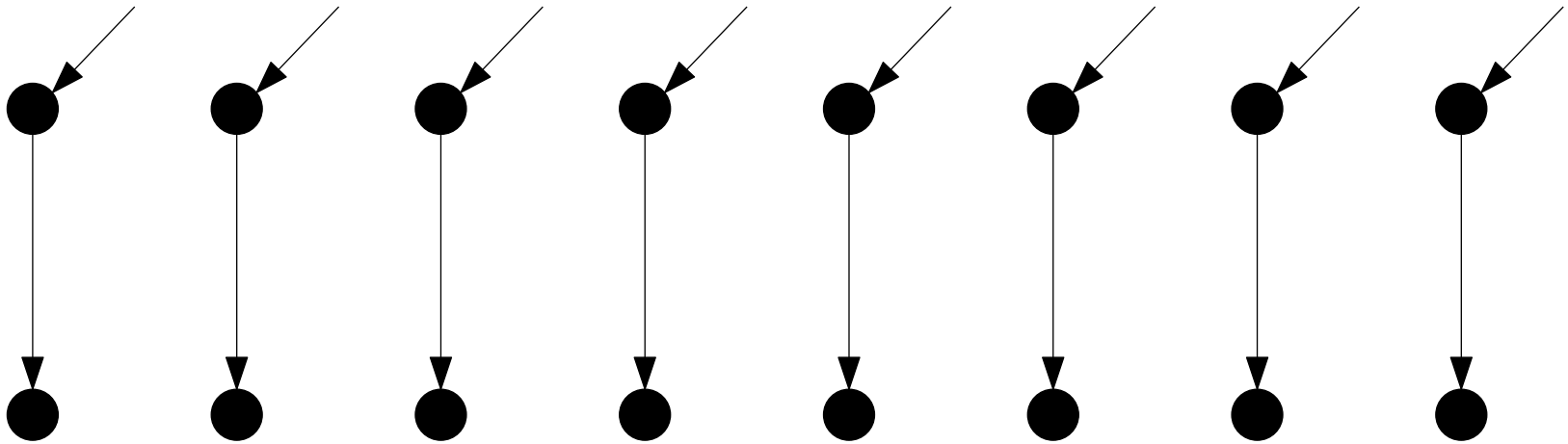

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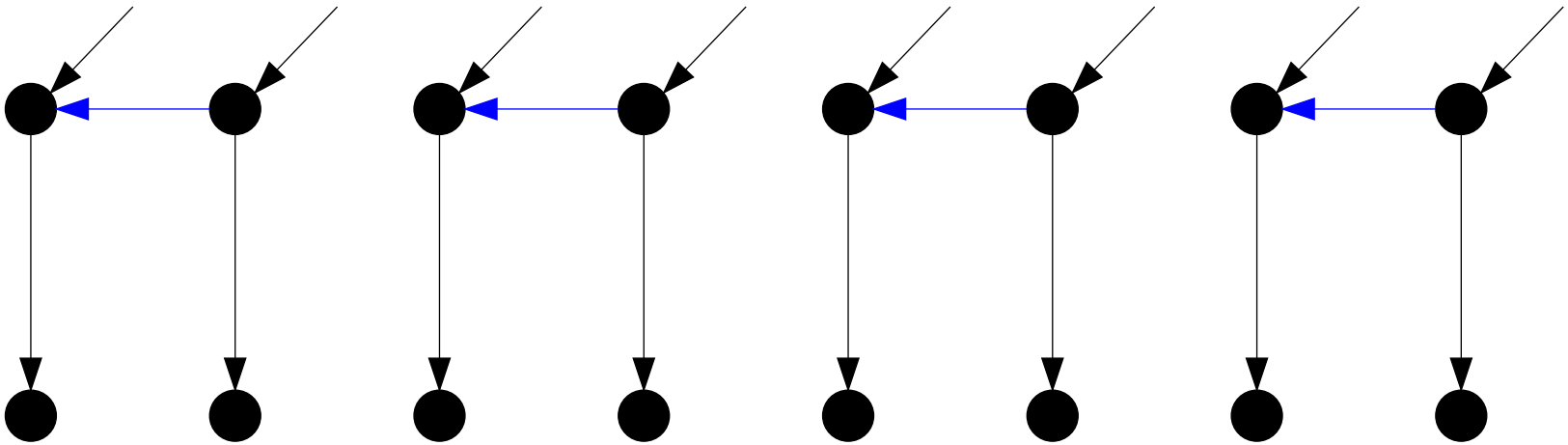

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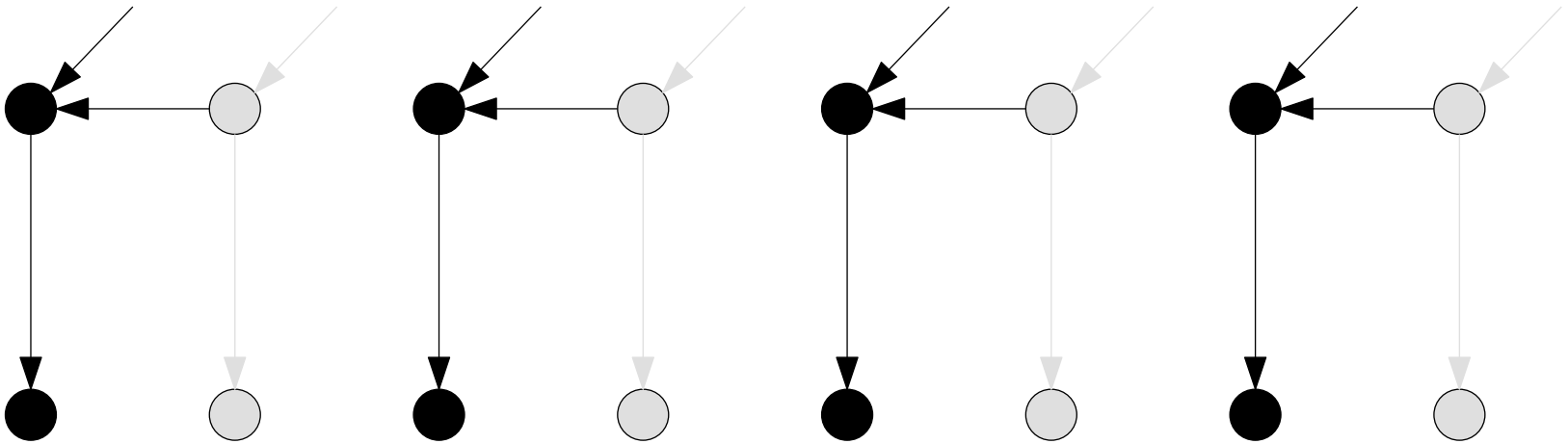

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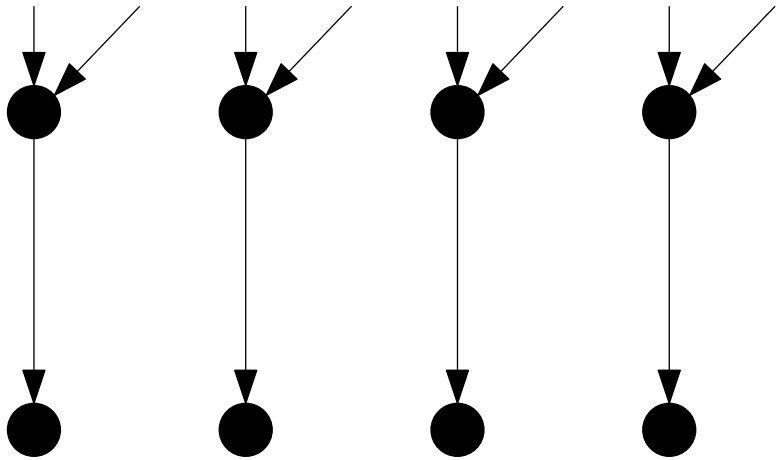

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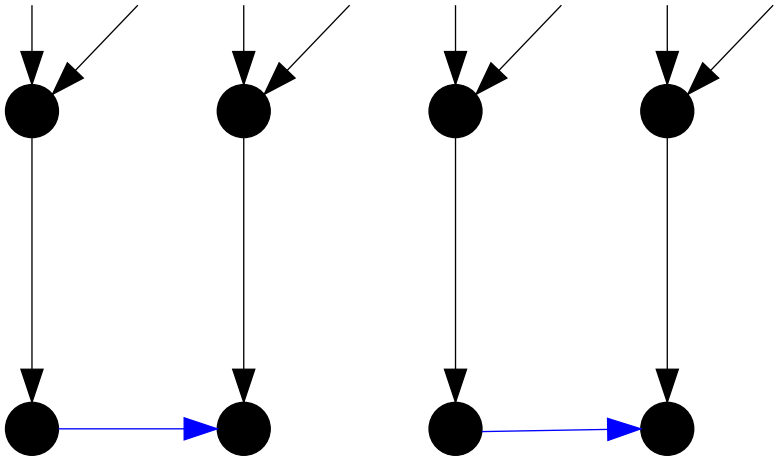

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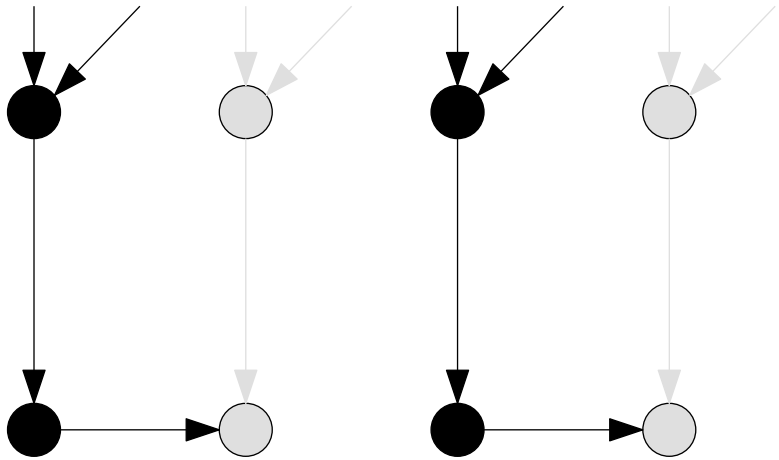

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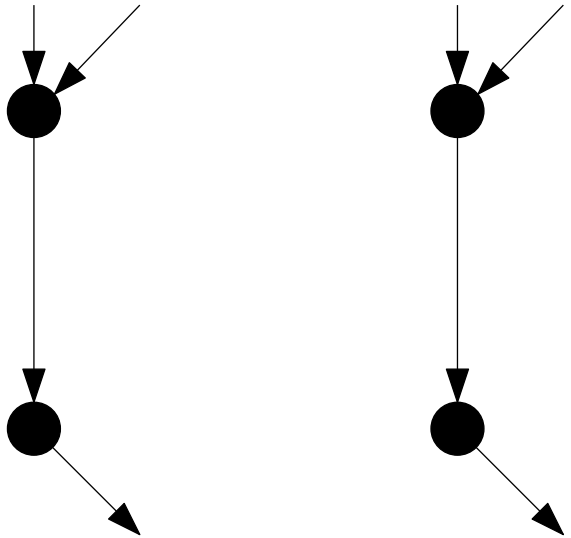

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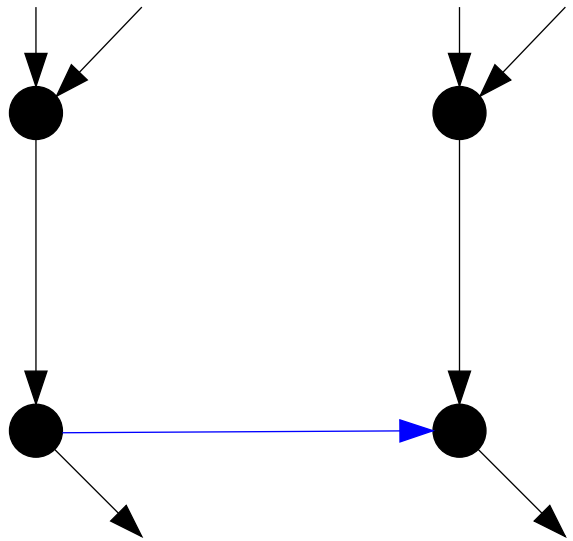

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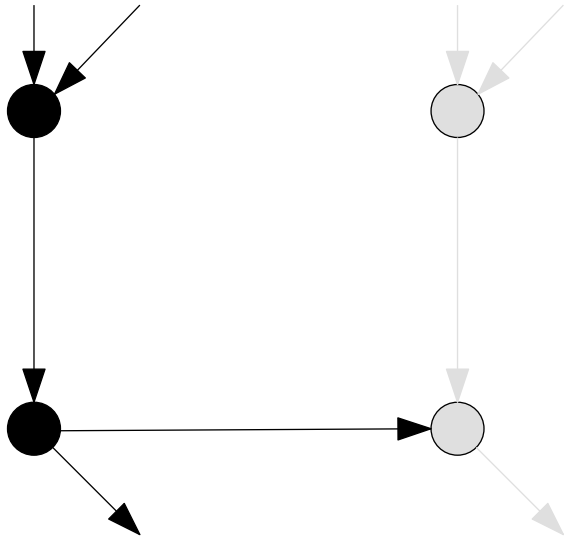

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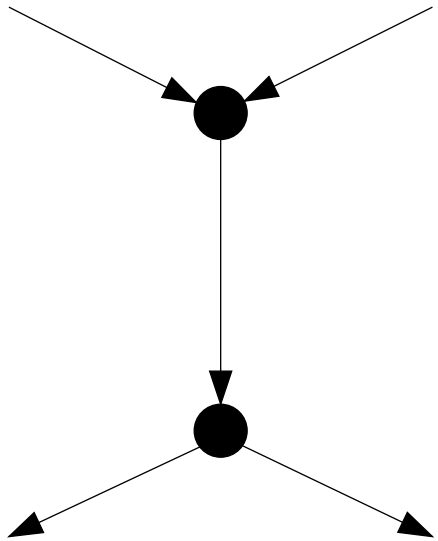

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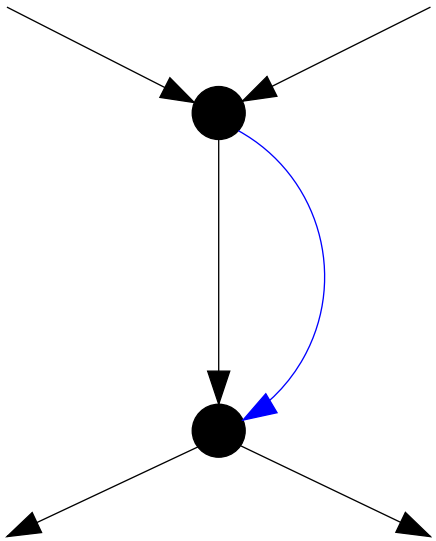

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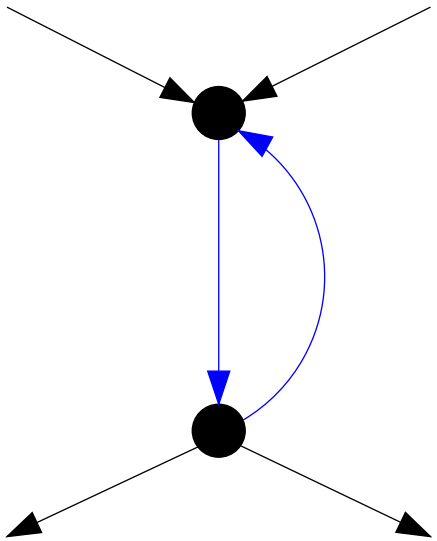

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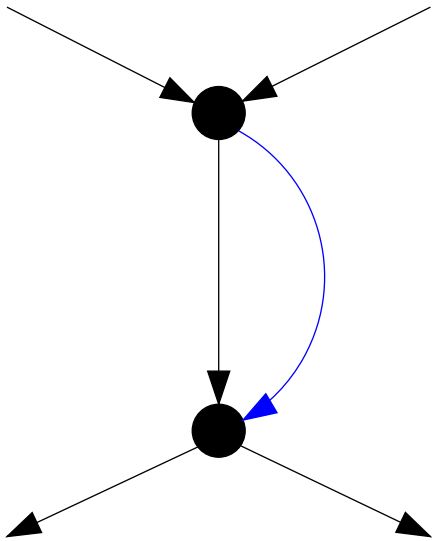

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$$c(x,y) = \begin{cases} 2^{-5} & \text{if } x = y \text{ and have not played yet,} \\ 0 & \text{if } x \text{ was not the first opponent of } y, \\ 2^{losses(x)+wins(y)-4} & \text{if } x \text{ beat } y \text{ in their first game,} \\ & \text{and } wins(x) = 1 \text{ and } losses(y) = 1. \end{cases}$$

Examples. If in their first game x beats y: $c(x,y) = 2^{-4}$.
If later x loses twice and y wins twice, then $c(x,y) = 1$.

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Where is the truth between 1.5 and 10?

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