Decomposition of Geometric Set Systems and Graphs

Dömötör Pálvölgyi

EPFL

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This presentation consists of two parts.

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I. Decomposition of Multiple Coverings.

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II. Slope Number.

Part I: Decomposition of Multiple Coverings

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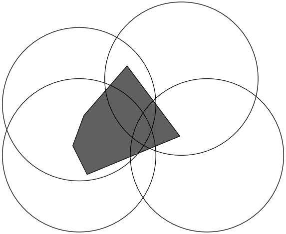

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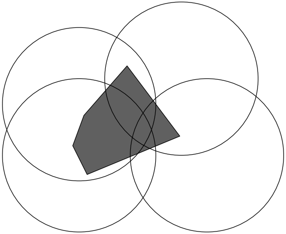

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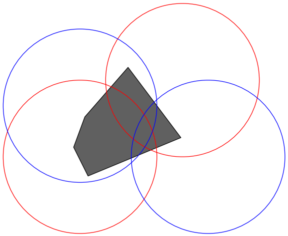

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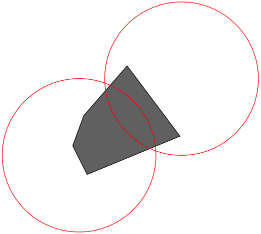

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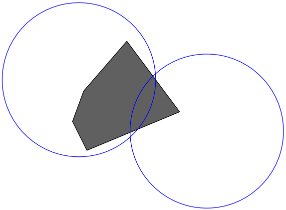

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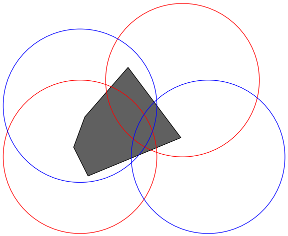

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From these, complete classification of polygons with respect to totally-cover-decomposability.

Deciding plane-cover-decomposability seems harder.

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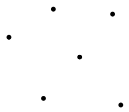

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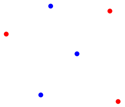

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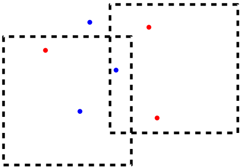

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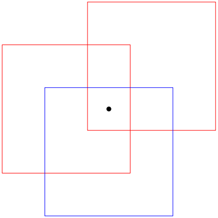

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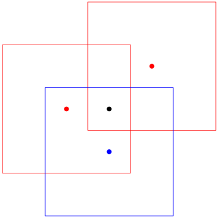

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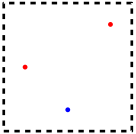

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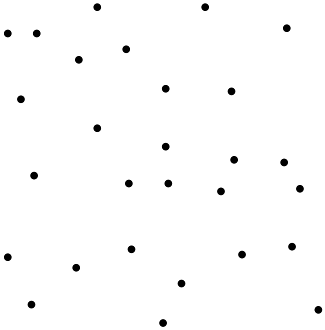

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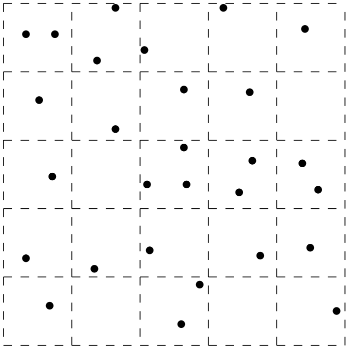

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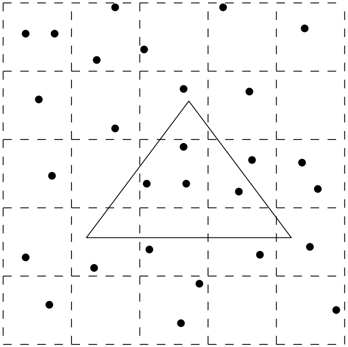

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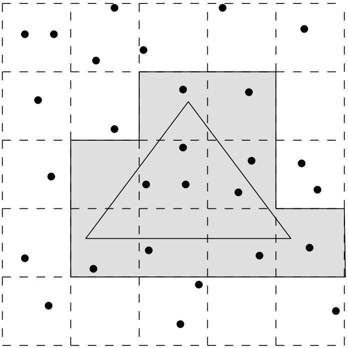

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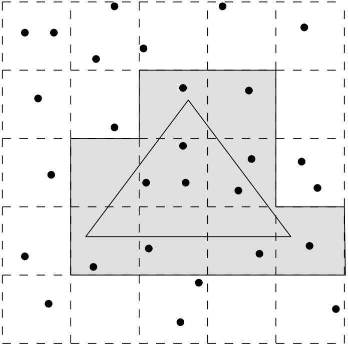

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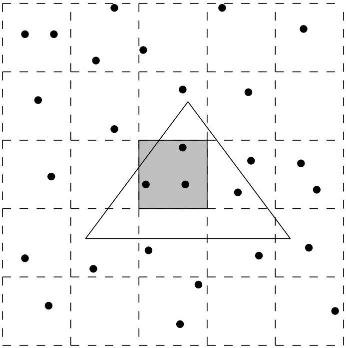

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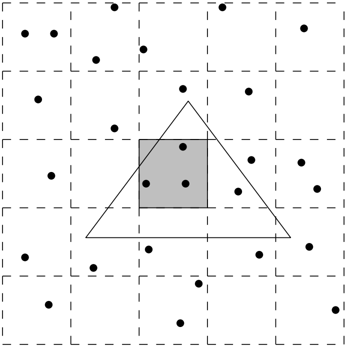

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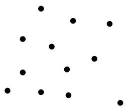

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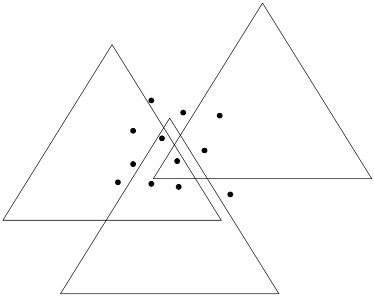

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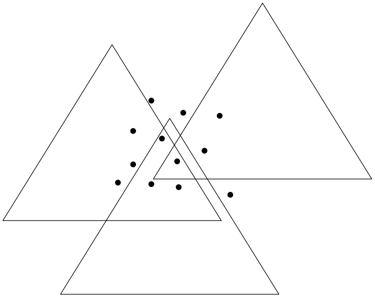

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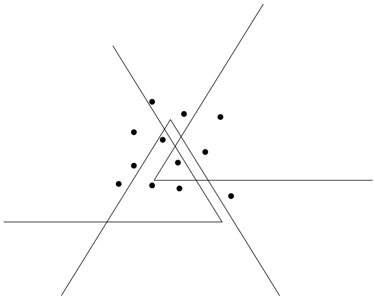

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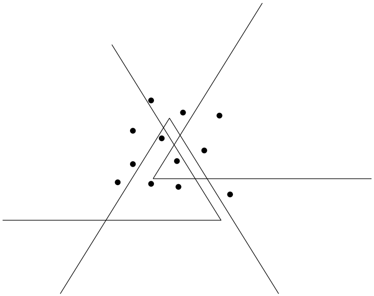

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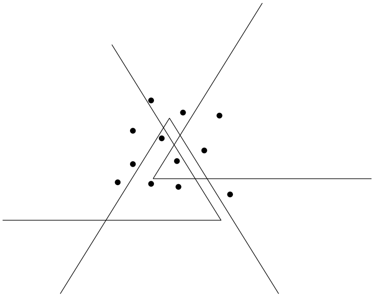

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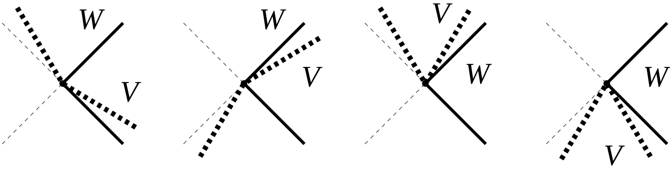

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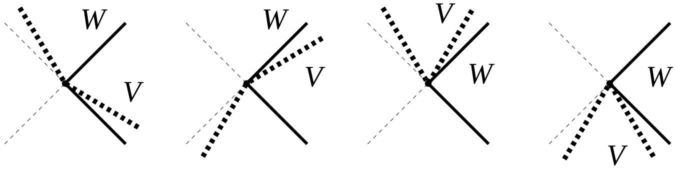
That is, the union of the wedges is in an open halfplane whose boundary contains the origin, but none of them contain the other.



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For any pair of special wedges V, W and for every k, l, there is a point set of cardinality $\binom{k+l}{k}$ such that for every coloring of the point set with red and blue either there is a translate of V containing k red and no blue points or there is a translate of W containing l blue and no red points.

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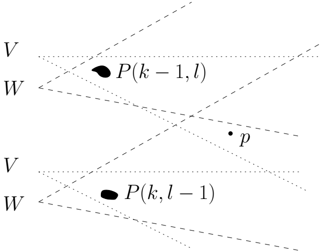

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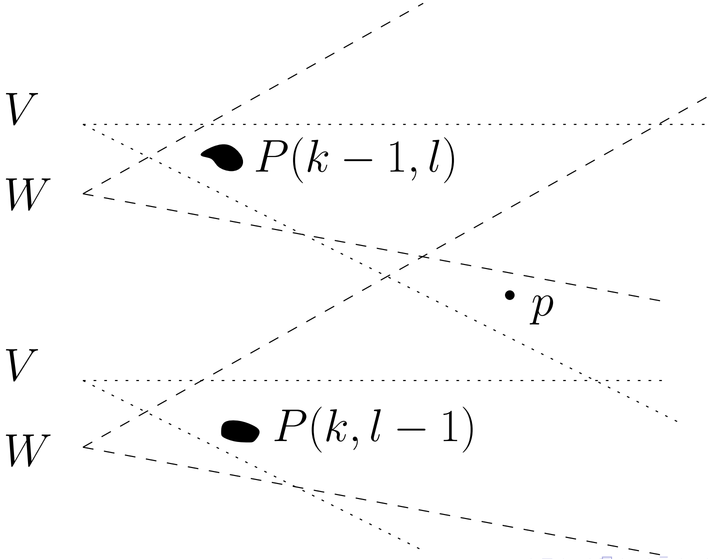

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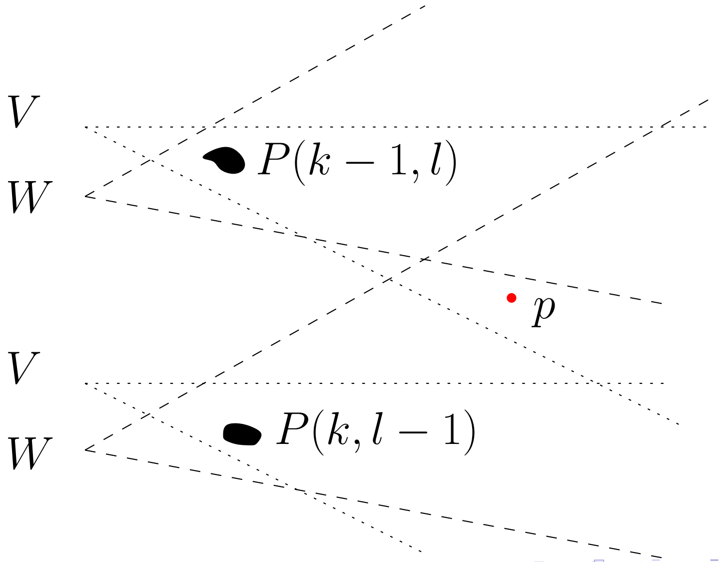

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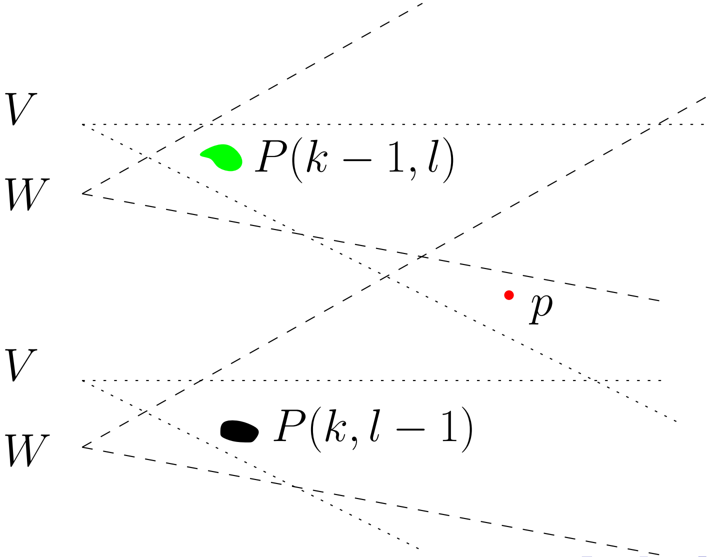

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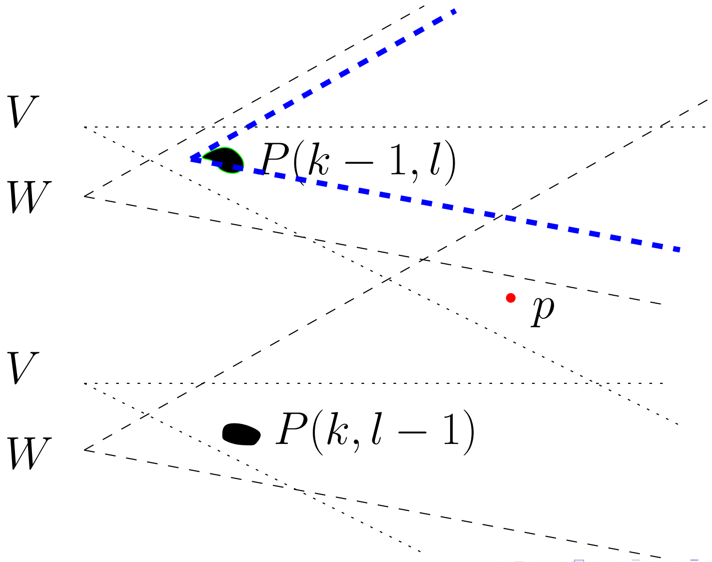

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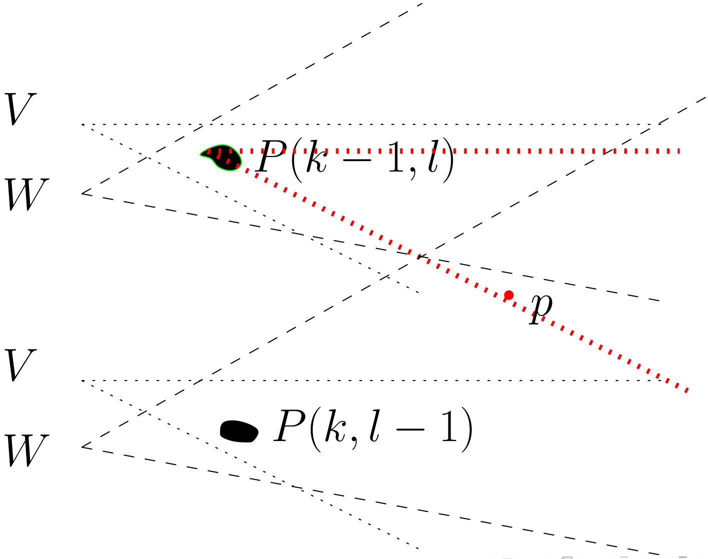

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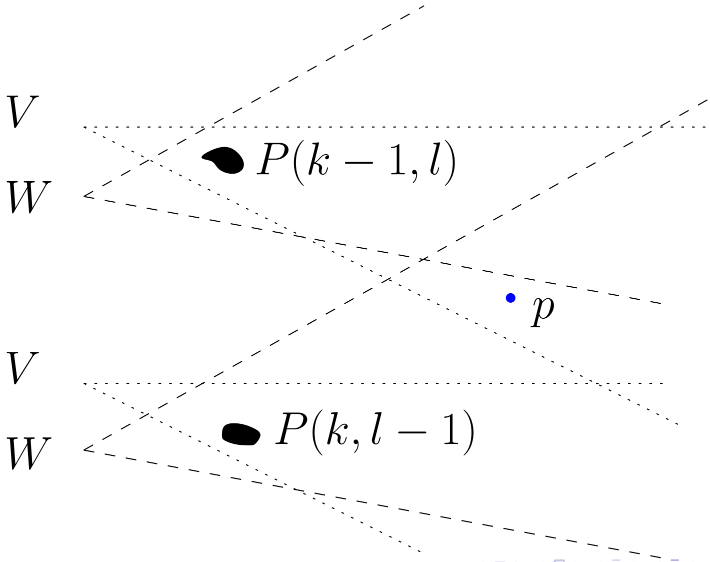

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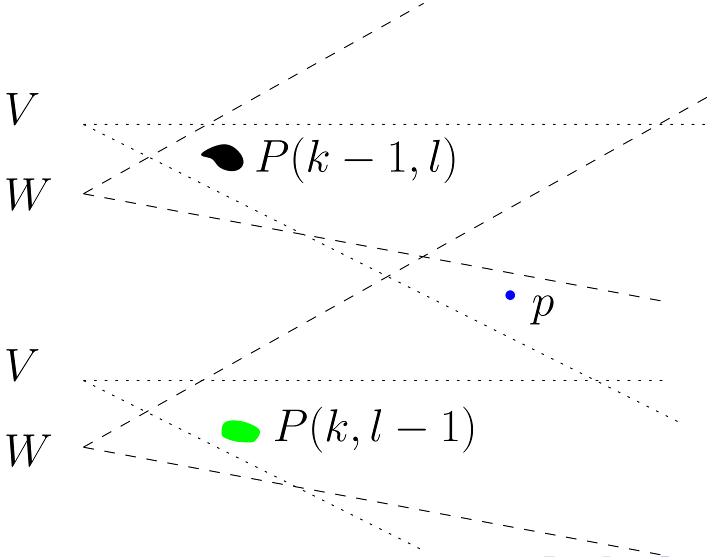

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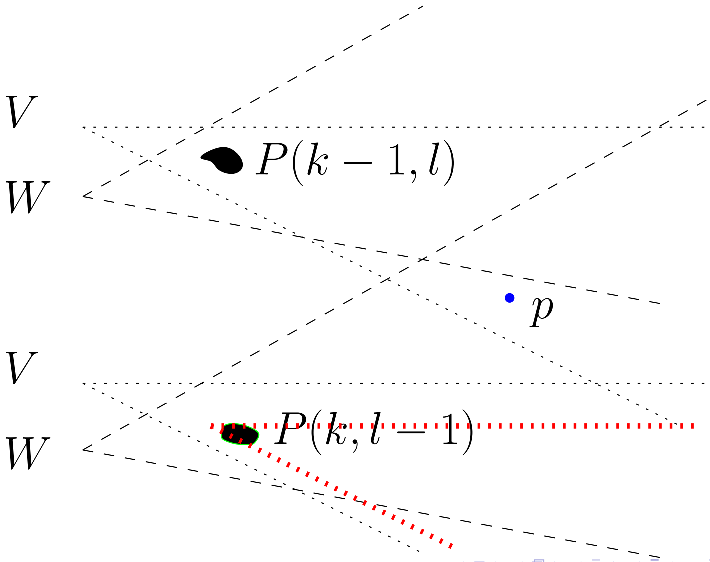

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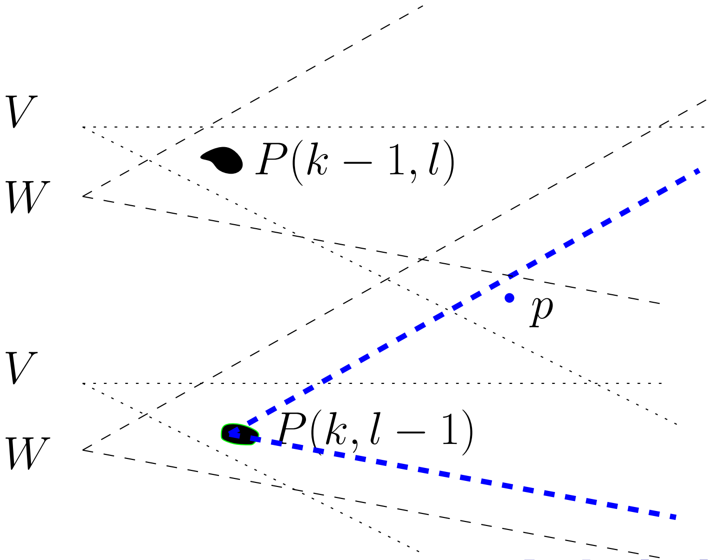

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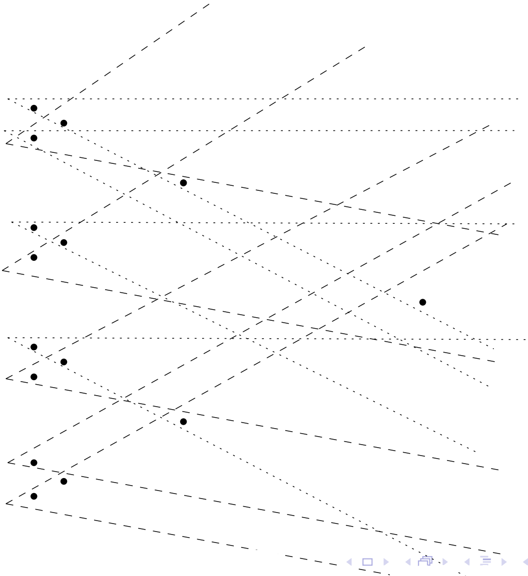

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A polygon is totally-cover-decomposable IFF it has no angles that form a special pair.

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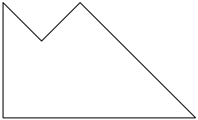

Figure: Cover-decomposable but not a finite union of translates of a convex set.

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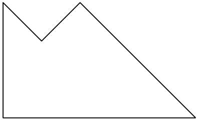

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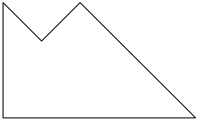

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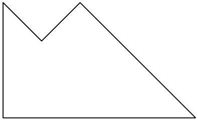

Figure: Cover-decomposable but not a finite union of translates of a convex set.

Corollary

Convex polygons are cover-decomposable.

Theorem

Polyhedrons are not cover-decomposable.
Not even space-cover-decomposable.

Decomposition to Multiple parts

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Part II: Slope Number

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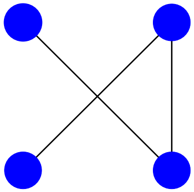
Example



A straight-line drawing of P_4.

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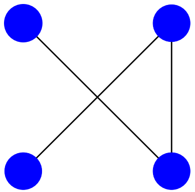

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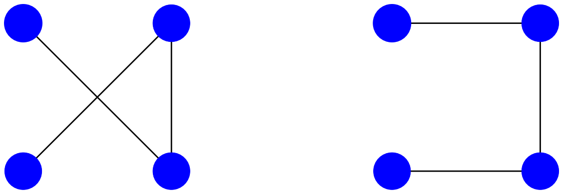

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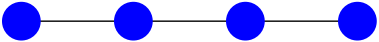

The slope number of P_4 is one.

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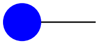

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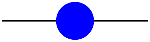

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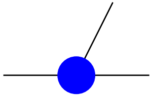

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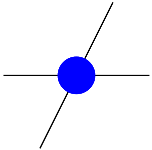

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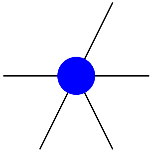

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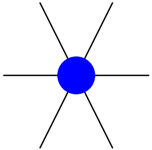

Question: Bounding slope number from above by a function of the maximum degree?

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Even for graphs with maximum degree five, the slope number can be arbitrarily large.

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The case of maximum degree four remains open.

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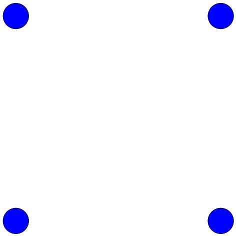

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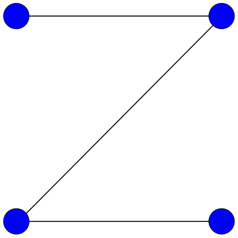

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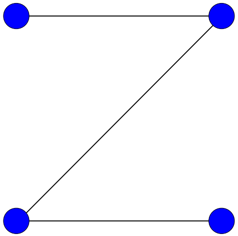
The slope parameter of P_4 is 2.



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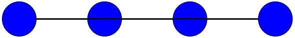

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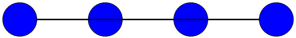

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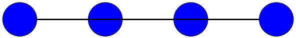

The slope parameter of K_4 is 1.

But for triangle free graphs always bigger than slope number.

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Degree four case is still open.

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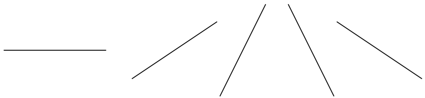

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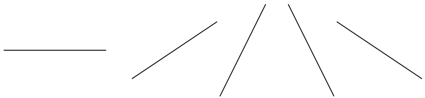

Theorem

Every subcubic graph has slope parameter at most five. *Moreover, this can be realized by a drawing such that no* three *vertices are collinear and each edge has one of the five basic slopes.*

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Then the omitted part is carefully reattached.

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Proposition

The degree of each vertex is at least two and G is two-connected.

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This can be done since G is 2-connected.

Note that there are no edges inside C because it is a shortest cycle passing through v.

Now let us delete C from the graph and using induction, take a drawing of $G' := G \setminus C$.

Imagine that G' is very small and put back C in a suitable way.

Putting Back $C = \{u_0, u_1, \ldots, u_4\}$

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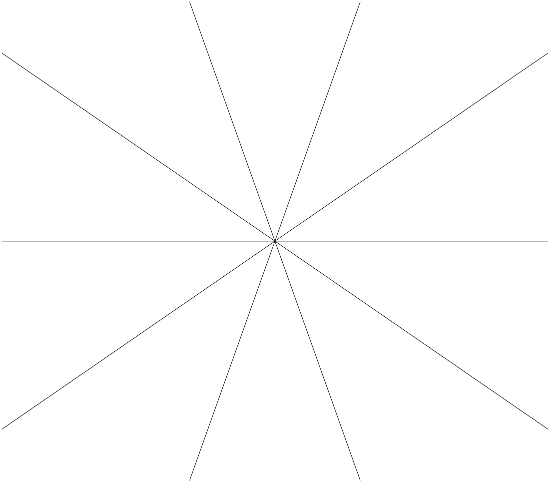
Take the five basic directions through origin.



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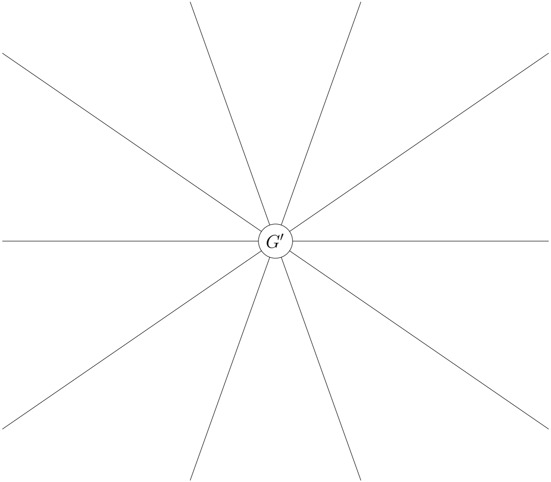
Put a small copy of G' into the middle.



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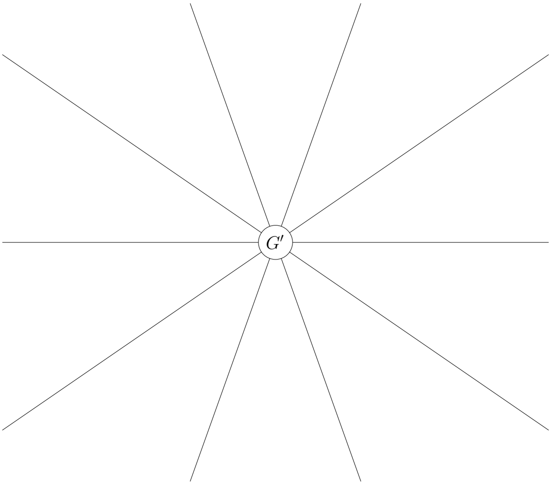
Find a place for u_1.



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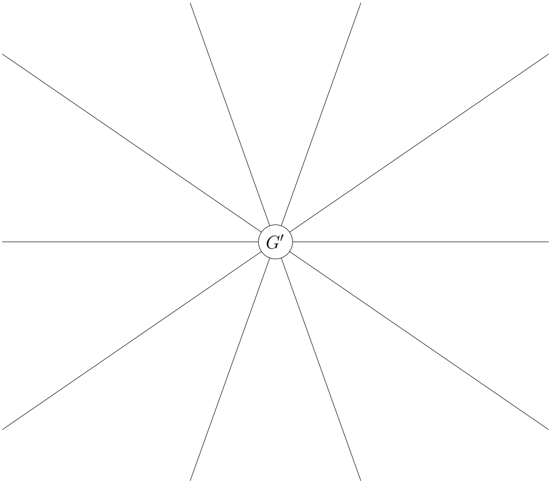
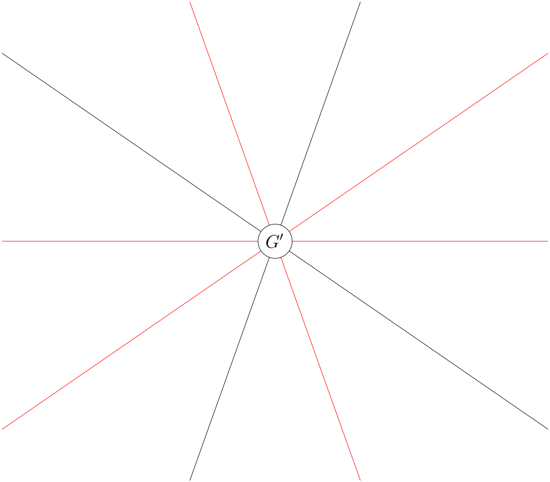
The neighbor of u_1 from G' can have at most two neighbors in G'.



Putting Back $C = \{u_0, u_1, \dots, u_4\}$

Therefore it has three *free* directions.



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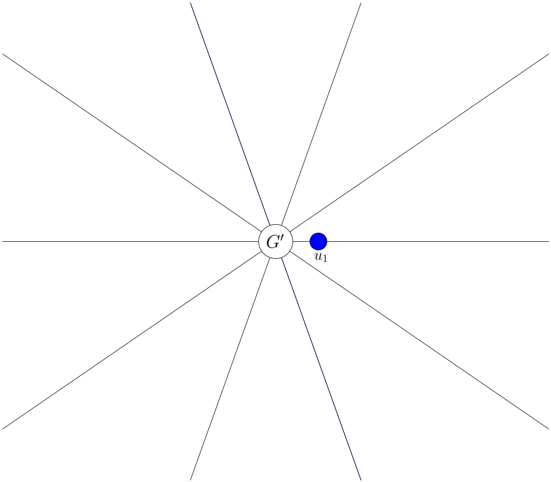
Place u_1 on one of them.



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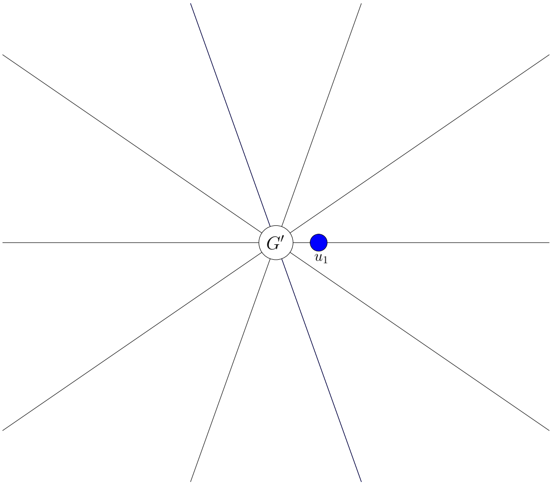
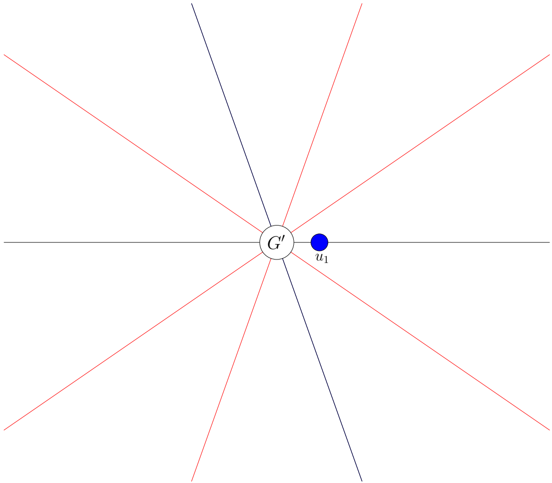
Find a place for u_2.



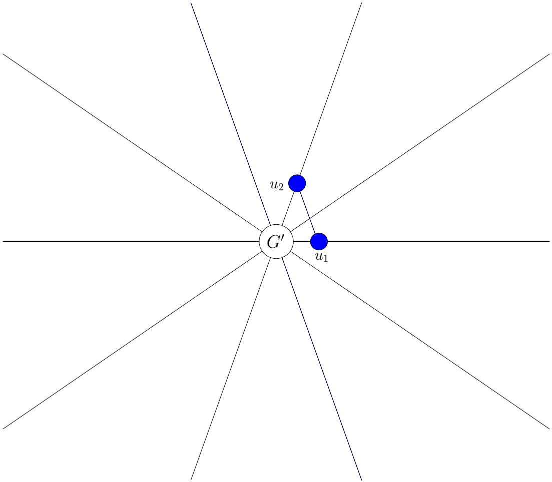
Putting Back $C = \{u_0, u_1, \ldots, u_4\}$

The neighbor of u_2 has three free directions.



Putting Back $C = \{u_0, u_1, \ldots, u_4\}$

Place u_2 on one of them that differs from the line of u_1.



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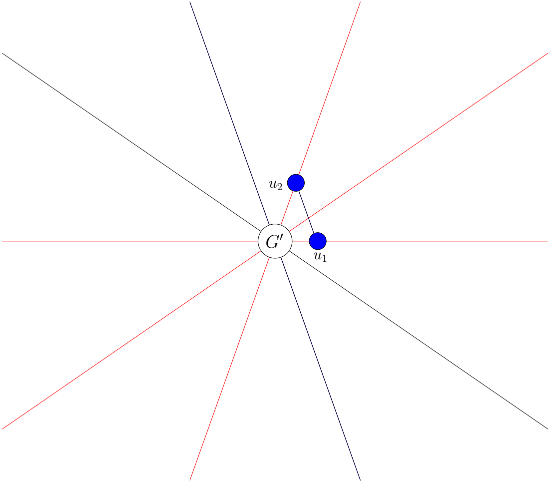
The neighbor of u_3 has three free directions.



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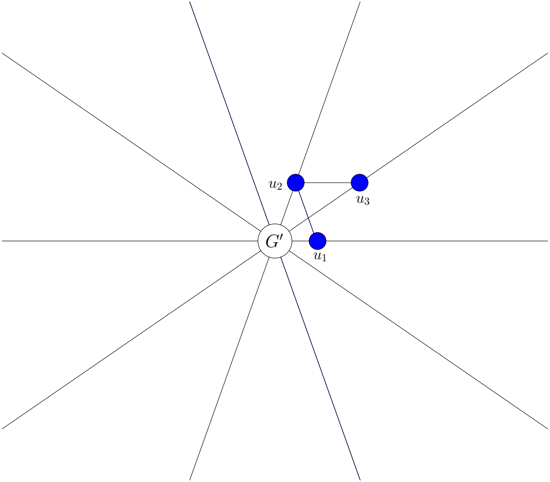

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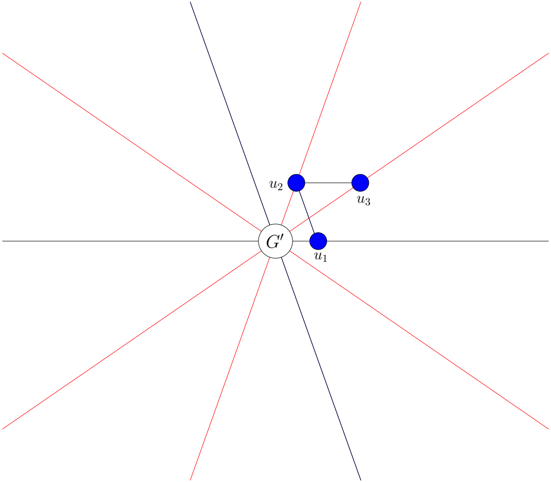

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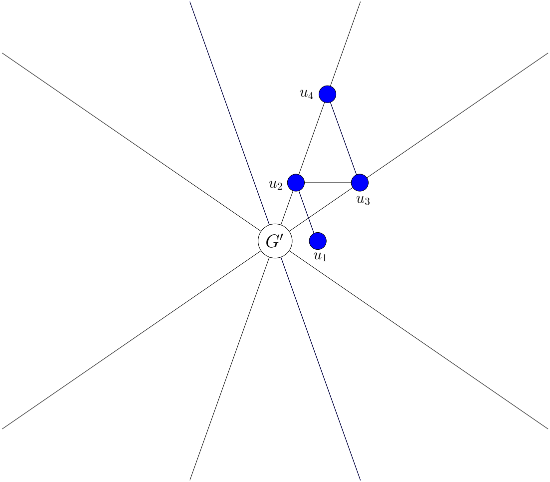
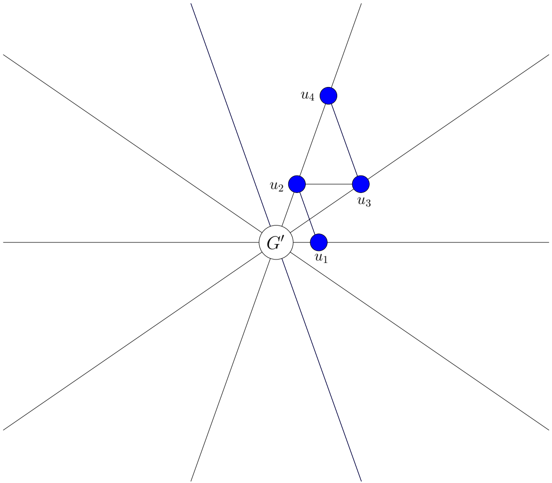
Place u_4 on one of them that differs from the line of u_3.



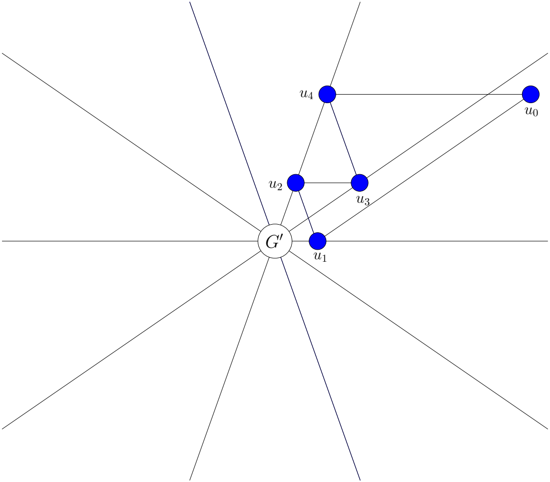
Putting Back $C = \{u_0, u_1, \dots, u_4\}$

Find a place for u_0.



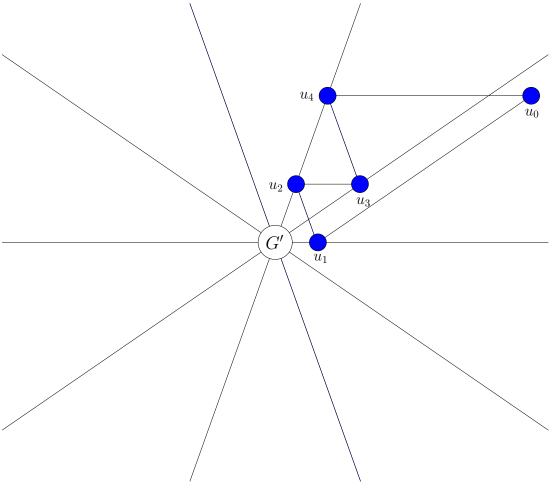
Putting Back $C = \{u_0, u_1, \ldots, u_4\}$

Tricky but can be done if its line does not neighbor the line of u_1.



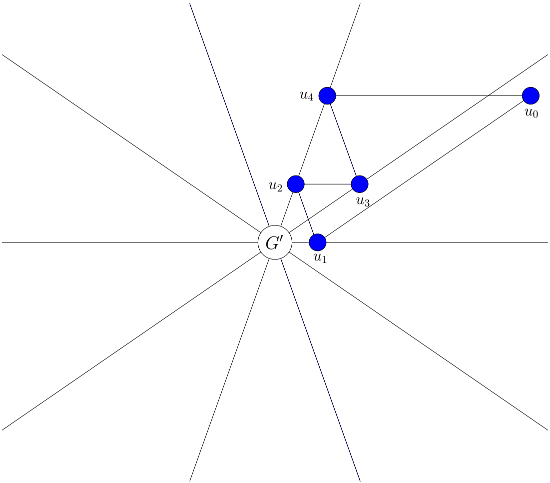
Putting Back $C = \{u_0, u_1, \ldots, u_4\}$

This can be achived by using the freedom that we had earlier.



Putting Back $C = \{u_0, u_1, \ldots, u_4\}$

This finishes the proof of the subcubic theorem.



Proof for Cubic Graphs

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Every cubic graph has slope parameter at most seven.

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Draw rest of graph using theorem for subcubic graphs.

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Proof for non-connected graph is very technical.

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Defined only for planar graphs.

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Note.
The number of slopes used is exponential in the maximum degree.

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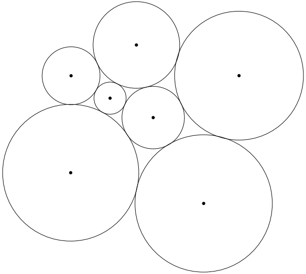

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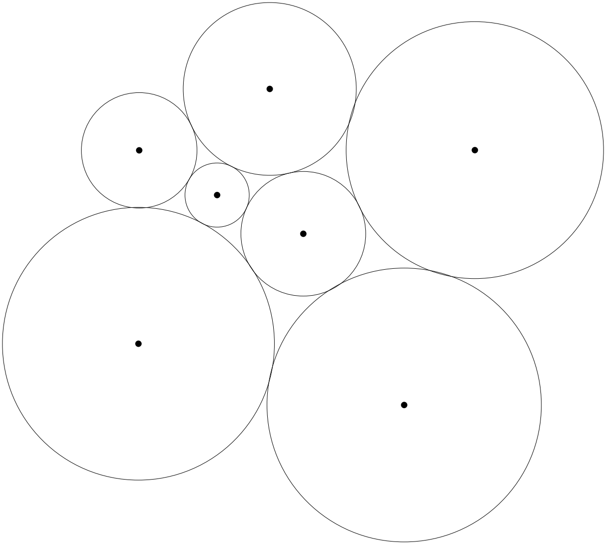

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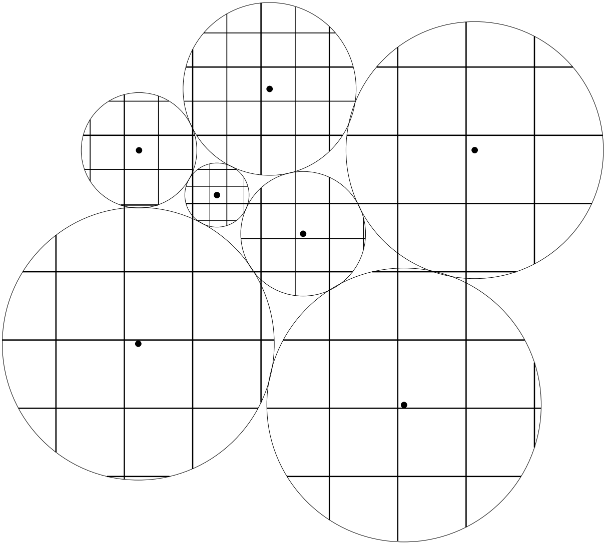

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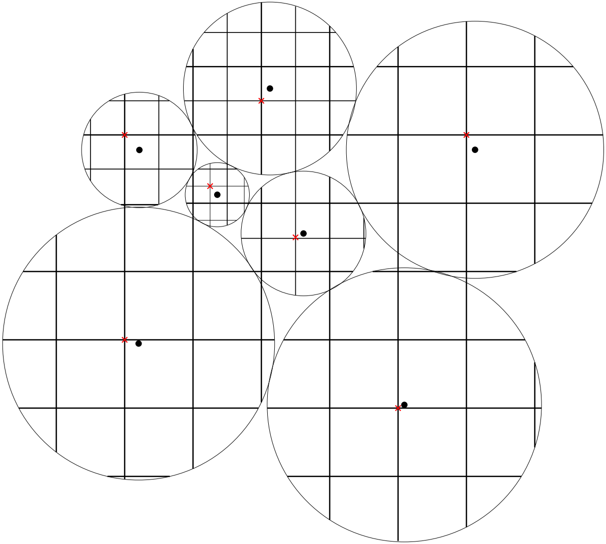

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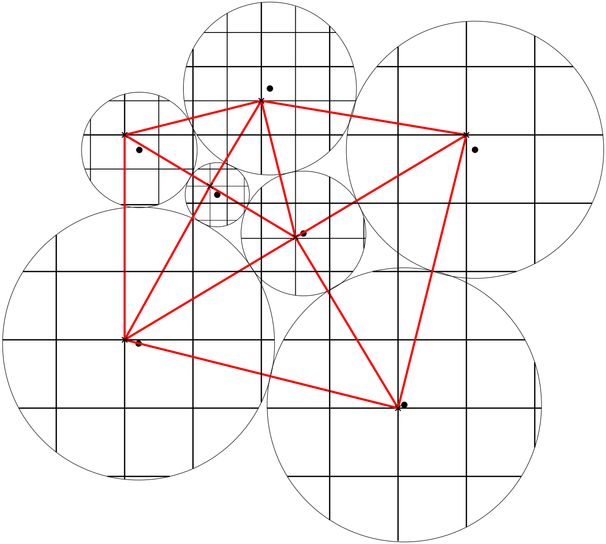

Thank you for your attention!