1. Describe the dual of a trivial representation.

2. Prove that if the representation T' is irreducible, then so is T.

3. Prove that $(R+S)' \simeq R' + S'$ for any two representations R and S of a group G.

4. Prove that if the representation T is completely reducible, then so is T'.

5. Prove that the identity representation of $\mathrm{SL}_2(K)$ is isomorphic to its dual.

6. Let $T: G \to \mathrm{GL}(V)$ be a completely reducible representation, and let $U \subset V$ be an invariant subspace. Show that $T \simeq T_U + T_{V/U}$.

7. Let T_1, T_2, and S be completely reducible finite-dimensional linear representations. Show that if $T_1 + S \simeq T_2 + S$, then $T_1 \simeq T_2$.

8. Prove that $(T_1 + T_2)S \simeq T_1 S + T_2 S$ for any representations T_1, T_2, and S of G.

9. Prove that $TS \simeq ST$ for any representations T and S of G.

10. Describe the square of a representation in terms of matrices.

11.* Let V and U be complex vector spaces, and let $\alpha \in \mathrm{L}(V)$, $\beta \in \mathrm{L}(U)$. The product of the representations $t \mapsto e^{t\alpha}$ and $t \mapsto e^{t\beta}$ of $\mathbf{C}$ is necessarily of the form $t \mapsto e^{t\gamma}$, where $\gamma \in \mathrm{L}(V \otimes U)$. Find the operator γ.

12. Let T and S be an irreducible representation and a one-dimensional representation, respectively, of the group G. Show that TS is irreducible.

13. Prove formula (9) without resorting to the matrix interpretation.

14. Interpret the representation $T \otimes T$ in terms of matrices, and compare it with T^2.

15. Prove that the complexification of any odd-dimensional irreducible real representation is irreducible.

16. Find all finite-dimensional representations of O_n whose kernels contain SO_n.

17. Find all one-dimensional representations of the group A_4.

18.* Prove that the commutator subgroup of $\mathrm{GL}_n(\mathbf{R})$ is equal to $\mathrm{SL}_n(\mathbf{R})$.