

### Homework 3.

*Submission deadline:* Monday, Oct 15, on paper or by email.

**1** Let  $S$  be a set of  $n$  points in the plane. Let  $a$  denote the minimum distance between two points in  $S$ . Construct a graph with node set  $S$ , by connecting two of them if their distance is  $a$ . Prove that this graph is planar.

**2** Let  $L$  be a set of  $n$  lines in the plane such that no 3 are concurrent. Construct a graph whose nodes are the intersection points of lines in  $L$ , and connect two of them if they are consecutive intersection points on one of the lines. (a) Prove that the graph obtained this has a node of degree two. (b) Prove that the graph is 3-colorable.

**3** (a) Let  $G$  be a planar map in which every face is an even cycle. Prove that the graph is bipartite.

(b) Let  $G$  be a planar map in which every face is a triangle and every node has even degree. Prove that the graph is 3-colorable.