Geometric Graph Theory
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Homework 6

Submission deadline: Monday, December 10.

1 (a) Let G be a graph on n nodes with minimum degree k. Prove that G has an orthonormal representation in $\mathbb{R}^{n-k+1}$.

(b) Suppose that a graph G has an orthonormal representation in $\mathbb{R}^{n-k+1}$ such that every set of $n-k+1$ nodes is represented by linearly independent vectors. Prove that G is k-connected.

2 Let G be the disjoint union of graphs G_1 and G_2. Prove that $\vartheta(G) = \vartheta(G_1) + \vartheta(G_2)$.

3 Prove that if G has an orthonormal representation in $\mathbb{R}^d$, then $\vartheta(G) \leq d$.

4 Let G_1 and G_2 be two perfect graphs.

(a) Form the disjoint union of G_1 and G_2, and connect every node of G_1 to every node of G_2. Prove that the resulting graph is perfect.

(b) Let B_i be a complete subgraph of G_i such that $|V(B_1)| = |V(B_2)|$. Form the disjoint union of G_1 and G_2, and identify the nodes of B_1 with the nodes of B_2. Prove that the resulting graph is perfect.

(c) Is $G_1 \boxtimes G_2$ always perfect?