Homework 1.

Submission deadline: Friday, Nov 28.
Solutions can be submitted on paper or by email, in English or Hungarian.

1 (a) Find the volume of the regular simplex in $\mathbb{R}^n$ with edges of length 1.

(b) Find the radii of the inscribed and circumscribed balls, and compare their volumes with the volume of the simplex.

2 Let K be a convex body in $\mathbb{R}^n$, and let a be its center of gravity.

(a) Let E be a line through a, and let u, v be the intersection points of E with the boundary of K. Prove that a must be in the middle $(n-1)/(n+1)$ part of the segment uv.

(b) Let H be a hyperplane through a. Prove that the volume of K on each side of H is at least a fraction of $1/e$ of the total volume (here $e = 2.71828182\ldots$).

3 Let K be a convex body in $\mathbb{R}^n$, and S be a simplex contained in K with maximum volume. Prove that that K can be covered by a congruent copy of nS.

Show how to use this to find an approximation of the volume of K, with a multiplicative error of n^{2n}.

4 Assign to each side b of a convex polygon P the maximum area of a triangle that has b as a side and is contained in P. Show that the sum of the areas is at least twice the area of P.