

## Selected topics in graph theory

Homework # 3

Date: May 13, 2017 Due: May 22, 2017

*(The solution can be submitted on paper or electronically.*

*I recommend pdf compiled from latex.)*

3.1 PROBLEM. Let  $K_{n,n}$  denote the the complete bipartite graph with  $n$  nodes in each bipartition class. Prove that for every graphon  $W$ ,

$$t(K_{n,n}, W) \geq t(K_2, W)^{n^2}.$$

3.2 PROBLEM. Let  $K'_r$  denote the graph obtained by deleting an edge from the complete graph  $K_r$  on  $r$  nodes. Prove that for every graphon  $W$ ,

$$t(K'_{r+1}, W) \geq \frac{t(K_r, W)^2}{t(K_{r-1}, W)}.$$

3.3 PROBLEM. We call a graphon  $W$  *bipartite*, if there is a partition  $[0, 1] = V_1 \cup V_2$  such that  $W(x_1, x_2) = 0$  for almost all  $(x_1, x_2) \in V_1 \times V_2$ . Prove that a graphon is bipartite if and only if  $t(C_{2k+1}, W) = 0$  for all  $k \geq 1$ . (Here  $C_n$  denotes the cycle on  $n$  nodes.)

3.4 PROBLEM. Prove that for any three simple graphs  $F, G, H$ , we have

$$t_{\text{inj}}(F, H) \geq t_{\text{inj}}(F, G)t_{\text{inj}}(G, H).$$

3.5 PROBLEM. Let  $F$  and  $G$  be two simple graphs, and let  $W$  be a graphon such that  $t(F, G) > 0$  and  $t(G, W) > 0$ . Prove that  $t(F, W) > 0$ . [Hint: Use the Lebesgue Density Theorem.]