

2. DIFFERENCIÁLSZÁMÍTÁS

25. FELADAT MEGOLDÁSOK

Számítsuk ki az alábbi függvények deriváltjait!

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| <p>(1) $f(x) = 5x^3 - 4x^2 + 3,$
$f'(x) = 15x^2 - 8x;$</p> <p>(2) $f(x) = \frac{x^2-x}{4},$
$f'(x) = \frac{x}{2} - \frac{1}{4};$</p> <p>(3) $f(x) = \frac{x-3}{x-5},$
$f'(x) = -\frac{2}{(x-5)^2};$</p> <p>(4) $f(x) = x + \frac{1}{x},$
$f'(x) = 1 - \frac{1}{x^2};$</p> <p>(5) $f(x) = \frac{2x+1}{x^{10}},$
$f'(x) = \frac{-18x-10}{x^{11}};$</p> <p>(6) $f(x) = \frac{1}{2} \frac{x^2-1}{x^2+1},$
$f'(x) = \frac{2x}{(x^2+1)^2};$</p> <p>(7) $f(x) = \sqrt{x},$
$f'(x) = \frac{1}{2\sqrt{x}};$</p> <p>(8) $f(x) = \sqrt[3]{x^2},$
$f'(x) = \frac{2}{3\sqrt[3]{x}};$</p> <p>(9) $f(x) = \sqrt{\frac{1}{x}},$
$f'(x) = -\frac{1}{2\sqrt{x^3}};$</p> <p>(10) $f(x) = \sqrt{\frac{1}{x} \sqrt[3]{x}},$
$f'(x) = -\frac{1}{4\sqrt[3]{x}};$</p> <p>(11) $f(x) = \sqrt{x \sqrt{\frac{1}{x} \sqrt{x}}},$
$f'(x) = \frac{3}{8x^{\frac{5}{8}}};$</p> <p>(12) $f(x) = \sqrt{x} \sqrt[3]{\frac{1}{x}} \sqrt{\frac{1}{x^3}},$
$f'(x) = -\frac{4}{3x^{\frac{3}{2}}};$</p> <p>(13) $f(x) = (x^2 + 1) e^x,$
$f'(x) = (x + 1)^2 \cdot e^x;$</p> <p>(14) $f(x) = x \sin x,$
$f'(x) = \sin x + x \cos x;$</p> <p>(15) $f(x) = e^x \sin x,$
$f'(x) = e^x (\sin x + \cos x);$</p> <p>(16) $f(x) = \ln x \cos x,$
$f'(x) = \frac{\cos x}{x} - \ln x \cdot \sin x;$</p> <p>(17) $f(x) = x (\ln x - 1),$
$f'(x) = \ln x;$</p> <p>(18) $f(x) = \sqrt{x} + \ln x - \frac{1}{\sqrt{x}},$
$f'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{x} + \frac{1}{2\sqrt{x^3}};$</p> | <p>(19) $f(x) = \frac{1}{\cos x},$
$f'(x) = \frac{\sin x}{\cos^2 x};$</p> <p>(20) $f(x) = \frac{1}{\ln x},$
$f'(x) = -\frac{1}{x \ln^2 x};$</p> <p>(21) $f(x) = \sin^5 x,$
$f'(x) = 5 \sin^4 x \cdot \cos x;$</p> <p>(22) $f(x) = \sin 5x,$
$f'(x) = 5 \cos 5x;$</p> <p>(23) $f(x) = \sin x^5,$
$f'(x) = \cos x^5 \cdot 5x^4;$</p> <p>(24) $f(x) = \sin^5 5x^5,$
$f'(x) = 125 \sin^4 5x^5 \cdot \cos 5x^5 \cdot x^4;$</p> <p>(25) $f(x) = \sqrt{\frac{1-x}{1+x}},$
$f'(x) = \frac{-1}{(1+x)\sqrt{1-x^2}};$</p> <p>(26) $f(x) = \frac{x}{\sqrt{1-x^2}},$
$f'(x) = \frac{1}{(1-x^2)^{\frac{3}{2}}};$</p> <p>(27) $f(x) = \frac{1}{(1+x^2)\sqrt{1+x^2}},$
$f'(x) = -\frac{3x}{(1+x^2)^{\frac{5}{2}}};$</p> <p>(28) $f(x) = \sqrt{x + \sqrt{x}},$
$f'(x) = \frac{1 + \frac{1}{2\sqrt{x}}}{2\sqrt{x + \sqrt{x}}};$</p> <p>(29) $f(x) = \sqrt[3]{(1-x)^2},$
$f'(x) = \frac{2}{3(x-1)^{\frac{1}{3}}};$</p> <p>(30) $f(x) = \ln \sqrt{\frac{1-x}{1+x}},$
$f'(x) = \frac{1}{x^2-1};$</p> <p>(31) $f(x) = \ln \frac{\sqrt{1-x^2}+1}{x},$
$f'(x) = -\frac{1}{x\sqrt{1-x^2}};$</p> <p>(32) $f(x) = e^{10} \sin 5x,$
$f'(x) = 5e^{10} \cos 5x;$</p> <p>(33) $f(x) = \ln \sqrt{\frac{1-\sin x}{1+\sin x}},$
$f'(x) = -\frac{1}{\cos x};$</p> <p>(34) $f(x) = \ln \sin x,$
$f'(x) = \cot x;$</p> <p>(35) $f(x) = \ln \ln x,$
$f'(x) = \frac{1}{x \ln x};$</p> <p>(36) $f(x) = \ln \tan \left(\frac{x}{2} + \frac{\pi}{4} \right),$
$f'(x) = \frac{1}{\cos x};$</p> |
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- (37) $f(x) = \ln(x + \sqrt{x^2 + a^2})$,
 $f'(x) = \frac{1}{\sqrt{x^2 + a^2}}$;
- (38) $f(x) = \arcsin 2x$,
 $f'(x) = \frac{2}{\sqrt{1-4x^2}}$;
- (39) $f(x) = \arccos \frac{1}{x}$,
 $f'(x) = \frac{1}{x^2 \sqrt{1-\frac{1}{x^2}}}$;
- (40) $f(x) = \ln \lg x$,
 $f'(x) = \frac{1}{x \ln x}$;
- (41) $f(x) = \ln 10^x$,
 $f'(x) = \ln 10$;
- (42) $f(x) = \lg e^x$,
 $f'(x) = \lg e$;
- (43) $f(x) = x^{2x}$,
 $f'(x) = x^{2x} \cdot 2(1 + \ln x)$;
- (44) $f(x) = x^{\sin x}$,
 $f'(x) = x^{\sin x - 1} (x \cos x \ln x + \sin x)$;
- (45) $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$,
 $f'(x) = \frac{-x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$;
- (46) $f(x) = \frac{1}{\lg 2^x}$,
 $f'(x) = -\frac{\lg 2}{\lg^2 2^x}$;
- (47) $f(x) = -x e^{-x} e^{-e^{-x}}$,
 $f'(x) = e^{-e^{-x} - 2x} (e^x (-1 + x) - x)$;
- (48) $f(x) = \frac{\sin x}{x} \arctan x$,
 $f'(x) = \frac{(1+x^2) \arctan x (x \cos x - \sin x) + x \sin x}{x^2 + x^4}$;
- (49) $f(x) = \tan x + \tan^3 x + \frac{3}{5} \tan^5 x$,
 $f'(x) = \frac{1}{\cos^2 x} (1 + 3 \tan^2 x + 3 \tan^4 x)$;
- (50) $f(x) = \frac{1}{2} \ln \tan \frac{x}{2} - \frac{\cos x}{2 \sin^2 x}$,
 $f'(x) = \frac{1}{\sin^3 x}$;
- (51) $f(x) = \frac{1}{2a} \left(\ln \frac{\sqrt{a^2+x^2}}{a+x} - \frac{a}{a+x} \right)$,
 $f'(x) = \frac{x^2}{(a+x)^2 (a^2+x^2)}$;
- (52) $f(x) = \ln \frac{1+\sqrt{x^2+1}}{x} - \sqrt{x^2+1}$,
 $f'(x) = -\frac{\sqrt{1+x^2}}{x}$;
- (53) $f(x) = |x|$,
 $f'(x) = \begin{cases} -1, & x < 0; \\ 1, & x > 0. \end{cases}$;
- (54) $f(x) = ||x-2|-2|+1$,
 $f'(x) = \begin{cases} -1, & x \in (-\infty, 0) \cup (2, 4); \\ 1, & x \in (0, 2) \cup (4, +\infty). \end{cases}$