Polyhedral Combinatorics – Exam Topics, Spring 2021

- 1. TU matrices, laminar and cross-free hypergraphs. Inverse of TU matrix is TU. Matrix of laminar bi-set family.
- 2. TDI systems. Edmonds-Giles teorem, Cook Theorem. Examples: Lucchesi-Younger theorem, rooted k-connected subgraphs.
- 3. Perfect matching polyhedron, matching polyhedron, TDI-ness.
- 4. The separation problem for the matching polyhedron. Seymour's theorem on T-joins in bipartite graphs. Upper hull of T-joins.
- 5. Clutter, blocker, covering polyhedron. Upper hull of T-cuts. Anti-blocker, packing polyhedron, conform hypergraphs.
- 6. Replication lemma, characterization of perfect hypergraphs. Weak perfect graph theorem. Polyhedral characterization of perfect graphs.
- 7. Bessy-Thomassé theorem. The Colorful Carathéodory theorem (including the stronger version).
- 8. Polyhedral proofs of matroid min-max theorems: matroid intersection theorem, maximum weight common independent set. Discrete separation theorem. Greedy algorithm for base polyhedra.
- 9. Generalized polymatroids and totally dual laminar systems. G-polymatroids defined by intersecting paramodular pairs, base polyhedra defined by crossing submodular set functions. Characterization of non-emptiness.
- 10. Intersection with a box and a plank, strong linking property. Spanning trees with prescribed degrees on a stable node set. Rooted k-edge-connected and k-edge-connected orientations with lower and upper bounds on the in-degrees.