

Summary of publications since PhD defence

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10. The p -adic Hodge decomposition according to Beilinson (jt. with Tamás Szamuely), positively refereed at *Proc. of 2015 Salt Lake City AMS Summer Institute in Algebraic Geometry* (October 2016), arXiv:1606.01921

A detailed presentation of Beilinson's approach to p -adic Hodge theory.

9. Multivariable (φ, Γ) -modules and products of Galois groups, to appear (2017?) in *Math. Res. Letters*, arXiv:1603.04231

We show that the category of continuous representations of the d th direct power of the absolute Galois group of \mathbb{Q}_p on finite dimensional \mathbb{F}_p -vector spaces (resp. finitely generated \mathbb{Z}_p -modules, resp. finite dimensional \mathbb{Q}_p -vector spaces) is equivalent to the category of étale (φ, Γ) -modules over a d -variable Laurent-series ring over \mathbb{F}_p (resp. over \mathbb{Z}_p , resp. over \mathbb{Q}_p).

8. Multivariable (φ, Γ) -modules and smooth \mathfrak{o} -torsion representations, to appear (2017?) in *Selecta Mathematica*, arXiv:1511.01037, DOI:10.1007/s00029-016-0259-5

Let G be a \mathbb{Q}_p -split reductive group with connected centre and Borel subgroup $B = TN$. We construct a right exact functor D_Δ^\vee from the category of smooth modulo p^n representations of B to the category of projective limits of finitely generated étale (φ, Γ) -modules over a multivariable (indexed by the set of simple roots) commutative Laurent-series ring. These correspond to representations of a direct power of $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$ via an equivalence of categories. Parabolic induction from a subgroup $P = L_P N_P$ gives rise to a basechange from a Laurent-series ring in those variables with corresponding simple roots contained in the Levi component L_P . D_Δ^\vee is exact and yields finitely generated objects on the category SP_A of finite length representations with subquotients of principal series as Jordan-Hölder factors. Lifting the functor D_Δ^\vee to all (noncommuting) variables indexed by the positive roots allows us to construct a G -equivariant sheaf $\mathfrak{Y}_{\pi, \Delta}$ on G/B and a G -equivariant continuous map from the Pontryagin dual π^\vee of a smooth representation π of G to the global sections $\mathfrak{Y}_{\pi, \Delta}(G/B)$. We deduce that D_Δ^\vee is fully faithful on the full subcategory of SP_A with Jordan-Hölder factors isomorphic to irreducible principal series.

7. Links between generalized Montréal functors (jt. with Márton Erdélyi), to appear (2017?) in *Math. Zeitschrift*, arXiv:1412.5778

Let \mathfrak{o} be the ring of integers in a finite extension K/\mathbb{Q}_p and $G = \mathbf{G}(\mathbb{Q}_p)$ be the \mathbb{Q}_p -points of a \mathbb{Q}_p -split reductive group \mathbf{G} defined over \mathbb{Z}_p with connected centre and split Borel $\mathbf{B} = \mathbf{TN}$. We show that Breuil's pseudocompact (φ, Γ) -module $D_\xi^\vee(\pi)$ attached to a smooth \mathfrak{o} -torsion representation π of $B = \mathbf{B}(\mathbb{Q}_p)$ is isomorphic to the pseudocompact completion of the basechange $\mathcal{O}_\mathcal{E} \otimes_{\Lambda(N_0), \ell} D_{SV}(\pi)$ to Fontaine's ring (via a Whittaker functional $\ell: N_0 = \mathbf{N}(\mathbb{Z}_p) \rightarrow \mathbb{Z}_p$) of the étale hull $\widetilde{D_{SV}(\pi)}$ of $D_{SV}(\pi)$ defined by Schneider and Vignéras. Moreover, we construct a G -equivariant map from the

Pontryagin dual π^\vee to the global sections $\mathfrak{Y}(G/B)$ of the G -equivariant sheaf \mathfrak{Y} on G/B attached to a noncommutative multivariable version $D_{\xi, \ell, \infty}^\vee(\pi)$ of Breuil's $D_\xi^\vee(\pi)$ whenever π comes as the restriction to B of a smooth, admissible representation of G of finite length.

6. On twists of modules over noncommutative Iwasawa algebras (jt. with Somnath Jha and Tadashi Ochiai), *Algebra & Number Theory* **10**(3) (2016), 685–694.

It is well known that, for any finitely generated torsion module M over the Iwasawa algebra $\mathbb{Z}_p[[\Gamma]]$, where Γ is isomorphic to \mathbb{Z}_p , there exists a continuous p -adic character ρ of Γ such that, for every open subgroup U of Γ , the group of U -coinvariants $M(\rho)_U$ is finite; here $M(\rho)$ denotes the twist of M by ρ . This twisting lemma was already applied to study various arithmetic properties of Selmer groups and Galois cohomologies over a cyclotomic tower by Greenberg and Perrin-Riou. We prove a non commutative generalization of this twisting lemma replacing torsion modules over $\mathbb{Z}_p[[\Gamma]]$ by certain torsion modules over $\mathbb{Z}_p[[G]]$ with more general p -adic Lie group G .

5. Algebraic functional equations and completely faithful Selmer groups (jt. with Tibor Backhausz), *Int. Journal of Number Theory* **11**(4) (2015), 1233–1257.

Let E be an elliptic curve—defined over a number field K —without complex multiplication and with good ordinary reduction at all the primes above a rational prime $p \geq 5$. We construct a pairing on the dual p^∞ -Selmer group of E over any strongly admissible p -adic Lie extension K_∞/K under the assumption that it is a torsion module over the Iwasawa algebra of the Galois group $G = \text{Gal}(K_\infty/K)$. Under some mild additional hypotheses this gives an algebraic functional equation of the conjectured p -adic L -function. As an application we construct completely faithful Selmer groups in case the p -adic Lie extension is obtained by adjoining the p -power division points of another non-CM elliptic curve A .

4. From étale P_+ -representations to G -equivariant sheaves on G/P (jt. with Peter Schneider and Marie-France Vignéras), *Automorphic Forms and Galois Representations* Volume 2-ben, LMS Lecture Note Series **415** (2014), 248–366.

Let K/\mathbb{Q}_p be a finite extension with ring of integers \mathfrak{o} , let G be a connected reductive split \mathbb{Q}_p -group of Borel subgroup $P = TN$ and let α be a simple root of T in N . We associate to a finitely generated module D over the Fontaine ring over \mathfrak{o} endowed with a semilinear étale action of the monoid T_+ (acting on the Fontaine ring via α), a $G(\mathbb{Q}_p)$ -equivariant sheaf of \mathfrak{o} -modules on the compact space $G(\mathbb{Q}_p)/P(\mathbb{Q}_p)$. Our construction generalizes the representation $D \boxtimes \mathbb{P}^1$ of $GL(2, \mathbb{Q}_p)$ associated by Colmez to a (φ, Γ) -module D endowed with a character of \mathbb{Q}_p^* .

3. (φ, Γ) -modules over noncommutative overconvergent and Robba rings, *Algebra & Number Theory* **8**(1) (2014), 191–242.

We construct noncommutative multidimensional versions of overconvergent power series rings and Robba rings. We show that the category of étale (φ, Γ) -modules over certain completions of these rings are equivalent to the category of étale (φ, Γ) -modules over the corresponding classical overconvergent, resp. Robba rings (hence also to the category of p -adic Galois representations of \mathbb{Q}_p). Moreover, in the case of Robba rings, the

assumption of étaleness is not necessary, so there exists a notion of trianguline objects in this sense.

2. Generalized Robba rings, (with an appendix by P. Schneider), *Israel J. Math.* **191**(2) (2012), 817–887.

We prove that any projective coadmissible module over the locally analytic distribution algebra of a compact p -adic Lie group is finitely generated. In particular, the category of coadmissible modules does not have enough projectives. In the Appendix a “generalized Robba ring” for uniform pro- p groups is constructed which naturally contains the locally analytic distribution algebra as a subring. The construction uses the theory of generalized microlocalization of quasi-abelian normed algebras that is also developed there. We equip this generalized Robba ring with a self-dual locally convex topology extending the topology on the distribution algebra. This is used to show some results on coadmissible modules.

1. Exactness of the reduction on étale modules, *Journal of Algebra* **331** (2011), 400–415.

We prove the exactness of the reduction map from étale (φ, Γ) -modules over completed localized group rings of compact open subgroups of unipotent p -adic algebraic groups to usual étale (φ, Γ) -modules over Fontaine’s ring. This reduction map is a component of a functor from smooth p -power torsion representations of p -adic reductive groups (or more generally of Borel subgroups of these) to (φ, Γ) -modules. Therefore this gives evidence for this functor—which is intended as some kind of p -adic Langlands correspondence for reductive groups—to be exact. We also show that the corresponding higher Tor-functors vanish. Moreover, we give the example of the Steinberg representation as an illustration and show that it is acyclic for this functor to (φ, Γ) -modules whenever our reductive group is $\mathrm{GL}_{d+1}(\mathbb{Q}_p)$ for some $d \geq 1$.